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ABSTRACT

Mathematics and the use of mathematical thinking should be much more than what has been traditional school arithmetic. Much of the mathematical reasoning can be developed and experienced out of school, particularly in the home. This material is a teacher's guide designed to help parents support what is done with their children in class. End-of-the-year assessment material is presented. A total of 31 activities on the following concepts and skills are included: (1) ratio; (2) using operations; (3) problem solving; (4) measurement; (5) fractions; (6) decimals; (7) computations; (8) geometry; (9) word problem; (10) logic; (11) scientific notations; (12) use of calculators; (13) use of LOGO; and (14) probability. (YP)

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MEANINGFUL MATHEMATICS

LEVEL FIVE

TEACHER'S GUIDE
TO
LESSON PLANS

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TABLE OF CONTENTS

Introduction
Basic Facts Review
Thinking
Guess My Rule
Ratio and Proportion
Arithmetic Problem Solving at Level Five
Problem Solving: Checking Up
Some Problems for Group Solving and Discussion
Problems to Solve as a Group Effort
Problems to Determine Operations to Use
Problems to Use - Missing Information
Problems to Use - Using Appropriate Data
Verbal Addition and Subtraction Problems: Some Dificulties and Some Solutions
Verbal Multiplication and Division Problems: Some Difficulties and Some Solutions
Graphs
Measurement
Fractions
Decimals
Computation in Base Ten: Background Information
General Review of Computation
Geometry
Volume: Geoblocks
Relationship
Open Sentences: Concrete
Logic
Problems Involving Money
Exponents and Scientific Notation
Using LOGO
Using the Calculator
Some Calculator Exploratios for Students
Using Data
Taking a Chance

LEVEL FIVE

INTRODUCTION:

At this level most students will be capable of what Piaget described as concrete thinking operations - reversibility of thought, use of compensating processes, relating of symbols to their referents, and the discrimination of linear, area, volume, and weight factors.

Also at this level basic ideas developed K-4 will be reviewed through application of them. New topics, or topics having increased emphasis include:

............the integration of fraction ideas and place value in work with decimals
..completing open sentences
............use of exponential and scientific notation
............closure on multiplication and division
............graphing of two step operations
............extension of ratio to direct proportions
............areas and volumes of non-standard shapes
............problem solving that involves:
    -extraneous data
    -missing data
    -multiple answer problems
    -use of logic
    -use of fractions and decimals
............calculator activities
............introduction of recursion in LOGO

Lessons are arranged by topics. Topics should be mastered before moving to another. However, you must make deliberate attempts to relate new materials to previously mastered material.

Work with calculators at this level should occupy more time than paper and pencil computation activities.

Repeat lessons as needed. Repeat or alter examples given to meet special needs. A case in point might be using decimals or fractions in sample problems that use whole numbers.

Most lessons given ONE explanatory example. Most lessons will require you use more than the one before seat work is assigned.

At this level, number sentences to model real situations or verbally described situations is necessary. Just as necessary is translating number sentences into real and verbally described situations.

While emphasis is on mastery of each concept, it is a good idea to periodically build in an activity that reviews all concepts and skills previously learned. Once each week is a good schedule for elementary school. This can be done orally in the primary grades and with a written exercise after symbolic representations have been mastered. Since all operations arise from combining, separating, comparing and part whole relations, as soon as the operation symbols are understood, these operations should all be presented at the same time in the written review exercises.

It is easier to do this at the beginning levels with orally presented review activities.

It is axiomatic for good teaching that past learnings should ALWAYS be integrated into new learning activities.

Several commercial products are available to supplement the lessons provided. These include, by topic:

Calculator
  KEYSTROKES SERIES

Fractions
  FRACTION BARS WORKBOOKS I & II
  EVERYTHING'S COMING UP FRACTIONS

Tangrams
  TANGRAMATH

Geometry
  GEOBLOCKS AND GEOJACKETS
  GEOBOARD ACTIVITY SHEETS
  DOT PAPER GEOMETRY

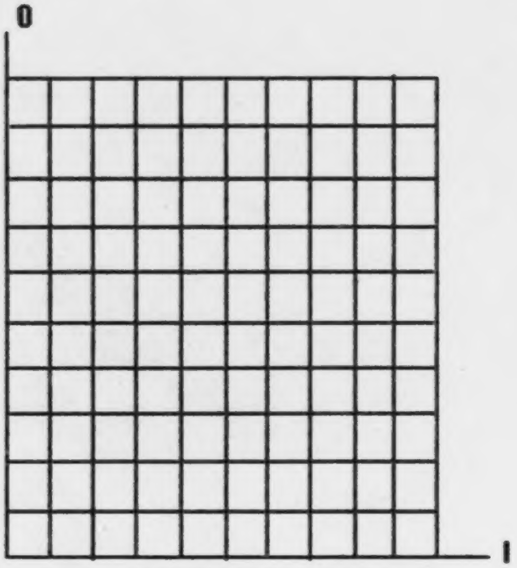
Logic
  ATTRIBUTE GAMES AND PROBLEMS
  ATTRIBUTE ACROBATICS
  DISCOVERING LOGIC
  ADVENTURES WITH LOGIC
  PLAYING WITH LOGIC
    (Cuisenaire)
  LOGIC IN EASY STEPS
  MINDBENDERS
    (Midwest Publications)

Problem Solving
  PROBLEM SOLVING EXPERIENCES IN MATHEMATICS
    (Addison Wesley)
  PROBLEM SOLVING IN MATHEMATICS - Lane County
    (Dale Seymour)

ASSESSMENTS TASKS (Level Five)

1. Give the student the following table of values and ask him/her to 1) find the equation to find O for any given I, and 2) graph the relationship on the graph paper supplied.

I	O
2	8
0	2
3	11
1	5
5	17
4	14



2. "For every 4 horses the circus has 3 large cats. The circus has 16 horses. How many large cats perform for the circus?"

3. Give the student 5 problems, - one requiring addition, one subtraction, one multiplication, one division and one requiring two of these four operations. (A sheet is attached for this and Task 4.)

4. Give the student a problem without some piece of information needed to find the answer.

5. Select an object in the room that is a uniform geometric shape. Have the student find the area of the largest face and the volume of the box, using a ruler to measure the dimensions.

6. Give the accompanying Fraction Test.

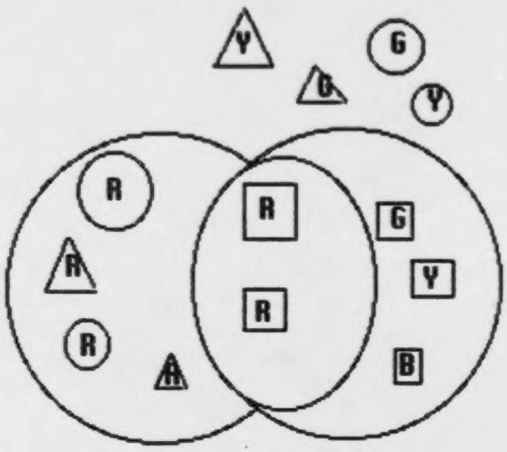
GEOMETRY: Show the student 3 different solid shapes - a cylinder, a triangular prism and a rectangular prism. The laatter two can be Geoblocks.

Ask the student to:

1. name each face
2. find the largest face
3. tell you how to find the area of each face
4. tell you how to find the volume in general
(area of base x height)

OPEN SENTENCES: A test is supplied.

LOGIC: Show the student the following array of attribute blocks:



"Point to the pieces that are RED AND SQUARE."

"Point to the pieces that are RED OR SQUARE."

"Point to the pieces that are NOT RED."

"Point to the pieces that are NOT SQUARE."

"Point to the pieces that are NOT (RED OR SQUARE)."

Remove the cards.

"Give me an if-then sentence that is true about the pieces you see."

"Remove a piece so that your sentence no longer will be true."

EXPONENTS
LOGO
CALCUATOR: Give the students the accompanying short test.

USING DATA: Give the students the accompanying problem.

TAKING A CHANCE:

"If I have 4 red marbles and 5 green marbles in a bag, mix it well and reach in and take a marble without looking, what is the probability of getting a red marble?"

"If I get a red marble and keep it, reach in the bag as before and take a second marble, what is the probability that it is red?"

"What is the probability of getting 2 red marbles in a row?"

LEVEL FIVE ASSESSMENT RECORD

Mathematician:____

LEVEL OF MASTERY

	Date:		Date:		Date:	
TOPIC	Progress Made	Mastery Attained	Progress Made	Mastery Attained	Progress Made	Mastery Attained
Finding Linear Relationships						
Ratio and						
Proportion						
Forming						
Using						
Using Operations						
addition						
subtraction						
multiplication						
division						
Mixed Operations						
Problems						
Finding Answer						
Recognizing Missing Data Needed						
Using Strategies						
Guess & Check						
Drawing Diagrams						
Making Tables						
Graphing						

LEVEL FIVE ASSESSMENT RECORD

LEVEL OF MASTERY

	Date:		Date:		Date:	
	Progress Made	Mastery Attained	Progress Made	Mastery Attained	Progress Made	Mastery Attained
Measurement						
Lengths						
Areas						
Volumes						
Fractions						
operations						
using operations in problems						
Decimals						
operations						
using operations in problems						
Computation in Base Ten						
paper & pencil						
Geometry						
areas						
volumes						
angles						

LEVEL FIVE ASSESSMENT RECORD

LEVEL OF MASTERY

	Date:		Date:		Date:	
	Progress Made	Mastery Attained	Progress Made	Mastery Attained	Progress Made	Mastery Attained
Open Sentences						
Solving w/one operation						
Solving w/two operations or more						
Logic						
Applications						
money						
distance/time						
others						
Exponents						
Scientific Notation						
Calculator Use						
Using Data						
Using LOGO						
Probability						

LEVEL FIVE ASSESSMENT WORKSHEET

Mathematician: ____________________

$2\square + 5 = 13$

$\square =$

$3\square - 5 = 16$

$\square =$

$2\square + 6 = 4$

$\square =$

$5\square + 1 = {}^{-}14$

$\square =$

LEVEL FIVE ASSESSMENT WORKSHEET

Mathematician:____

1. The roduct of 2^5 and 2^3 is ________________.

2. Written in scientific notation, 435.7 is ____________________.

3. DRAW
   The result of following this LOGO program. Put an X where the turtle is at the end of the program

```
TO BOXES
 REPEAT 2 [REPEAT4[FD 40 RT 90] FD 40]
END
```

4. Show how the calculator keys should be pushed to do this calculation:
   (4.5 x 13) + (284 ÷ 12)

LEVEL FIVE ASSESSMENT WORKSHEET

Mathematician:________________________________

"For the given data set, I (1) Put the data in order; (2) Found the range; (3) Found the mean; (4) Graphed this information on the beam."

DATA:

30,15,29,21,12

RANGE

MEAN:

Label the Hi, Lo and mean below the beam.

Mathematician:____

FRACTION TEST

12 3/4 ÷ 2 2/3 = ☐

☐ = 1 5/8 + 1 1/2 + 2 3/4

5/8 of 13 1/2 is ☐

☐ = 2 1/2 - 1 7/8

Bill cut a board 12 ft. long into pieces 2' 8" long. How many pieces did he get? How long was the piece left over?

	ft.	in.
☐	☐	☐
no. of pieces	pieces left over	

WORK

Tammy needs 3 pieces of ribbon 2' 4" long. How much ribbon should she ask the clerk for?

ft.	in.
☐	☐

WORK

Mathematician:____

FRACTION TEST

To make a shelf, Brenda needs 3 pieces. Two are 1 3/4 ft. long and the 3rd is 2 5/8 ft. long. If she allows 1/4" for each saw cut, how long a piece should she start with?

ft.	in.

Work:

Tom needs a piece of lumber 3 3/4 ft. long. He cuts it from a 6 ft. board. The saw uses 1/4". How long is the piece left?

ft.	in.

Work:

Mathematician:____________________________________

DECIMAL TEST

$$\begin{array}{r} 2.143 \\ \underline{+\ .005} \end{array} \qquad \begin{array}{r} 14.056 \\ \underline{+7.183} \end{array} \qquad \begin{array}{r} 29.041 \\ \underline{+16.991} \end{array}$$

$$\begin{array}{r} 3.000 \\ \underline{-1.832} \end{array} \qquad \begin{array}{r} 5.013 \\ -\underline{3.829} \end{array} \qquad \begin{array}{r} 19.878 \\ \underline{-9.439} \end{array}$$

52.1 x .43 =

19.8 x 1.04 =

19.8 ÷ 3.41 =

14.56 ÷ 2.07 =

Cashews were priced at 8.72 marks per kilogram. Tom asked the clerk for .4 kilograms. How many marks did he pay?

From this menu, Tessie ordered 2 items that cost less than the $5.00 she had. Which 3 could she order?

Hamburger	$2.19
Fishwich	$3.89
Chicken Sandwich	$3.98
French Fries	$1.39

Bob had 4.41 g. of a chemical in a jar. He carefully weighed out the 2.75 g. he needed for an experiment. How many g. of the chemical remained in the jar?

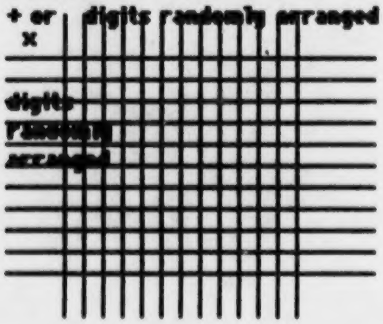
Jane helped her aunt at her whole foods store. She weighed out .60 oz. packets of spice from a jar labelled 3 pounds. How many packets could she fill? How many ounces were left in the jar when the last packet was filled?

LEVEL FIVE

BASIC FACTS REVIEW

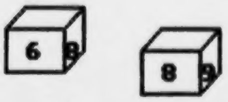
Periodically basic fact recall by:

1. Oral drills
2. Use of the following kinds of forms:



Use a different random order of digits each time.

3. Card games that use numbers
4. A dice pair with operation specified. Dice should have digits 0-5 on several, 6-11 on several. Mix the two kinds and specify the operation:



Multiply

5. Continue to have the class skip count and count on as a group
6. Stress the relationship between division and multiplication and between addition and subtraction. The 3 digit fact "families" such as: 3.4.12 3 x 4 = 12 4 = 12 3, 12 4 = 3 should be stressed
7. Emphasize:

-multiplication by 0 gives 0
-division by 0 makes no sense and can't be done

Background: These activities should be used for daily work for 6 or 7 weeks and then occasionally for maintenance the rest of the year.

In developing and maintaining mastery of multiplication facts, it is helpful to point out the patterns that exist:

- -the products of 2 are always even numbers
- -analyze the patterns in the products of each number nine and less

	3		4	
	6	no lead digit	8	sequence of even digits repeats
	9		12	
groupings of 3	12		16	
	15		20	
	18		24	
	21		28	
	24	lead digit 2	32	
	27		36	
	30		40	

5	end in alternating 5-0	6	
10		12	sequence of even digits repeats
15	lead digits cluster by 2 in order	18	
20		24	order is opposite to "4" order
25		30	
30		36	
35		42	
40		48	
45		54	
50		60	

7		8	even digits in reverse
14		16	order repeat
21	every digit used	24	lead digit repeats only
28	differences are	32	if decade number other-
35	33, 7, 33, 9, etc.	40	wise in order
42		48	
49		56	
56		64	
63		72	
70		80	

9	this is prettiest
18	1st digits in order
27	second digits in reverse
36	order
45	line of symmetry
54	for digit reversal
63	sum of digits is
72	always 9
81	
90	

This only highlights the need to search for patterns everywhere - number tables, such as addition and multiplication tables, hundreds chart, etc.

LEVEL FIVE

THINKING

Background: These activities should be used for daily work for 6 or 7 weeks and then occasionally for maintenance the rest of the year.

LESSON ONE: Getting Started (GS)

Children must know where to start in developing a plan of attack of a problem. This lesson presents several situations so children have to decide what to do first.

"You would like to buy a new bicycle."

"Where should your thinking about this start?"

List all suggestions given. Decide as a group on the order of thinking steps in resolving this. Some other GS situations to use:

"Sue wants to go to summer camp with her school friends, but her parents think she is too young."

"Should children who come to live in the U.S. be taught in English in school, or should the school offer classes in the children's language?"

LESSON TWO: Putting Things Together (PTT)

Children must learn to organize their information, the steps in some plan of action, or the materials in their school kits.

"Why are books organized into sections in the library?"

One of the advantages of organizing or classifying physical things is in the saving of time in finding things.

"How would you organize your thinking about how to deal with the few students in this class who continually interrupt the class?"

"How do you need to organize information obtained in connection with buying a bicycle?"

LESSON THREE: Zeroing In (ZI)

In considering possible approaches to a decision or facets of a situation, it often becomes draining of energy to distribute it over several possibilities at the same time - "too many irons in the fire." Children need to realize it is better to focus on ONE thing at a time in many situations. In doing in this, it is even more important to recognize WHAT is being considered.

"What is being looked at in the case of each question being asked?"

MAJOR TOPICS	QUESTIONS
School Rules	Some people don't care because they think they won't be caught
	It's fun to see how close you can get to breaking a rule without doing it
Working in School	Some people just don't like to take orders
	Some things are more interesting than others
	It's easier to say you didn't care to try than it is to admit failing
Teachers	Some teachers are crabby
	Mrs______ always says "Hello" when she sees me

LESSON FOUR: Drawing Conclusions (DC)

A lot of good thinking is done for nothing if there is no closure or end point. Children should realize conclusions are needed for discussions, processes, etc. Conclusions can take the form of:

.............ideas
.............answers
.............solutions
.............judgment
..........opinions
..........actions
..........images
..........values

Conclusions can be definite or tentative in nature. Once reached, however, they can be expressed in a clear way.

> "When a fourth grade class was asked if the school day should be shorter, they concluded this was a good idea. What about this conclusion?"

Consider in analysis:
...factors used to arrive at the conclusion
............consequences of the action
............data used that supported the conclusions

Situation: "Quality of cafeteria meals in the school was discussed."
Conclusion: "Schools waste money on school meals."
Have this conclusion analyzed

Situation: "How students get elected to school offices was discussed."
Conclusion: "School elections are popularity contests and are unrelated to the ability to do the job."
Analyze this conclusion.

LESSON FIVE: Where We Are Now (Consolidation)

Frequently it is desirable to collapse several related points of view or ideas into one or two more general views or ideas that include most of those started with. In group efforts, individual opinions often must be set aside in favor of a collective, or group, consensus.

"After a discussion on what makes a good teacher, a fifth grade class summarized as follows:

> You can tell right away. It depends on the subject. Good teachers make it interesting. Some teachers are boring. Good teachers don't have to be nice. Sometimes you don't learn anything from a nice teacher."

What is wrong with how this group consolidated its thinking?

SUBJECT	CONSOLIDATION
Homework	Some parents do the homework. I'd rather watch TV. Homework takes time. It's easier to copy someone else's.
Cheating	Everyone cheats sometimes. It's OK if you don't get caught Robbing a bank is worse
Grades	Not everyone tries for A's Teachers use grades to pick on certain kids. Everyone wants to know about grades. You need good grades to get into college.

Have students analyze each of these consolidating or summarizing statements. Do they clearly reflect the broadest ideas of the discussion? Do they represent an attempt to generalize? Have different ideas been put together into a single, brood statement?

LESSON SIX: What Is It? (Recognition)

What enables one to recognize a person, an object? What are the dangers of mistaken identity? Is recognition a guess?

Why is recognition and subsequent classification, important? What information about something automatically comes when we recognize what it is?

"We use properties of characteristics to make recognitions. Sometimes we can use more information before deciding. Sometimes we can make a guess and see if it works. Sometimes we can limit the possibilities to just a few."

Consider these:

"________has 4 legs." What possibilities are there?

"________also has horns." Does this help to reduce the possibilities?

"________has short hair." How about now?

"________has short legs." Are you getting closer to seeing what it is?

"Now what else would you like to know to be sure of what it is?"

Develop classification activities like this where children can work down through a hierarchy to get at finer descriptions and discriminations.

The VERBAL CLASSIFICATION books from Midwest Publications have activities that are a good source of ideas for these.

LESSON SEVEN: Analysis

Analysis in Bloom's Taxonomy is one step up from comprehension. It involves breaking a whole into its parts so these can be looked at individually and separately and the connections and relationships between these seen. These parts may be real or perceived. A bicycle has physical parts - wheels, frame, chain, etc. It also has perceived "features" that may be subjectively evaluated - speed, appearance, safety, etc.

Have the children divide the following into: real parts (rp) and perceived parts (pp):
School Home Football Team Boy/Girl Scouts

LESSON EIGHT: Comparing

Comparison is the basis for ordering, classifying, evaluating, and several other intellectual operations. We use comparison to distinguish things. When the number of similarities is high we look for differences. When the number of differences is high, we look for likenesses.

"These seem to be very different. Can you find ways in which any two or more are alike?"

Watching TV
Combing your hair
Going to school
Studying science
Brushing your teeth
Cop chasing robbers
Riding a bicycle

This should precipitate good group discussion of what could be some far fetched likenesses.

"Discuss these descriptions of someone who gossips a lot:
An old woman
A walking newspaper
A motor mouth."

"Compare spending money on technology such as a high speed Monorail train and on basic scientific research in research centers and universities."

LESSON NINE: Finding Alternatives

Some people think of a bottle as half empty, others as half full. A car is used for transportation. It is also a way of making a living for a car salesman.

"Give alternative descriptions or uses for:
a chicken
a book
a pencil
money
a test
a wagon
a uniform
school."

"Consider these situations. Suggest 2 alternative ways of approaching them:

A food supplement that is nutritious but tastes bad
As speeds go up, cars are in more accidents
Two boys are caught cheating on a test
Two girls are caught cheating on a test

LESSON TEN: Making Choices

In choosing something, a person has a set of conditions or requirements in mind, whether consciously or unconsciously. Children should learn to get these out in the open and state them.

"Which of the following is the best means of transportation?

- bicycle
- motorcycle
- all terrain vehicle
- snowmobile
- car
- bus
- train
- airplane."
- subway
- cable car
- moving sidewalk

Be sure the requirements that each best satisfied are thoroughly explored.

"If the school had a big fire and could not be used for
awhile, what buildings could be used to hold classes?"

"What are the personal qualities needed to be successful
at:

- being a teacher
- being a car salesman
- being a newspaper reporter
- being a TV anchor
- being a politician
- being a doctor?"

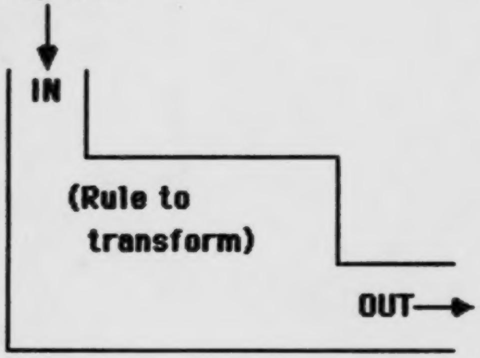
LEVEL FIVE

GUESS MY RULE

Children should have been introduced to this in LEVEL THREE. However, this cannot be assumed so it is best to review the rules of the game and to give some experience with simple rules.

The basic idea is that of a function machine that transforms numbers put in according to a fixed rule and generates numbers that come out. The result is a set of ordered pairs of numbers of the form:

(IN, OUT)



An example:

IN	RULE	OUT
2	+ 3	5
4	+ 3	7

LESSON ONE: How to Play the Game:

CHILDREN ARE TO RAISE HAND AND SAY, "I KNOW THE RULE, " WHEN THEY THINK THEY DO.

1. Children are to input numbers. Call on them one at a time.
2. For each number, you use a predetermined rule to orally give a number back to the class.
3. Repeat until a child raises his hand to test whether or not he/she knows the rule.

CHILDREN TEST BY SAYING, "IF I GIVE YOU (NUMBER), WILL YOU GIVE ME (NUMBER)?" THEY ARE NOT TO BLURT OUT THE RULE!

4. If you reply, "YES", the child is to keep silent and think of a better way to organize the information.

5. If "NO", remind all that they must always get further information if they cannot see the rule with what they have.
6. Periodically, ask those who think they have the rule to raise their hands.
7. Continue until:
    a. most have the rule; or
    b. they seem to be at a stalemate
8. Ask for tables of data and analyze. EXAMPLE:

DATA AS GENERATED

IN	OUT
2	6
5	9
1	5
3	7
10	14

Ask for suggestions as to what to do to make seeing patterns of numbers easier. Discuss: Encourage students to look for patterns in the OUT numbers after they have put the IN numbers in numerical order.

REORGANIZED DATA

IN	OUT
1	5
2	6
3	7
5	9
10	14

"How are the IN numbers changing?"

"How are the OUT numbers changing?"

"How do you get an OUT number for a given IN number?"

RULE: OUT = IN + 4

Have the students supply numbers to fill in any missing number pairs in an organized table.

LESSON TWO

Review the activity with some simple rules such as "add two", "multiply by three", "subtract five," "square it", etc.

Be sure students put the IN numbers in sequential order each time so that patterns are easier to see.

LESSON THREE

This introduces combined operation rules. Some examples are:
OUT = 3 X IN + 2

OUT = 2 x IN +5

OUT = 2 x IN - 1

We'll use 3 x IN + 1 as an example. When students arrange their data, it probably will result in a table like this:

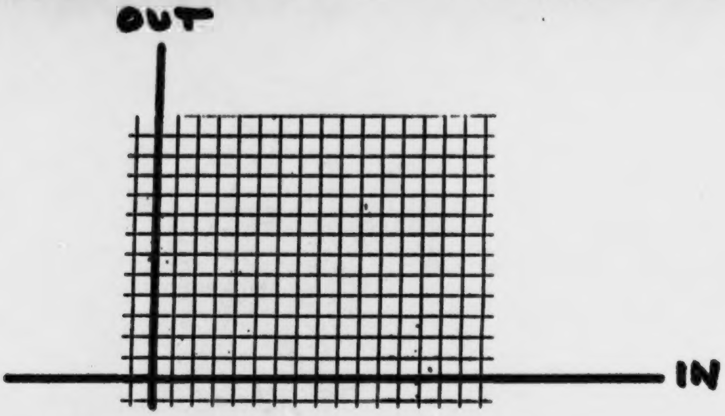
IN	OUT
0	1
1	4
2	7
3	10
4	13

Ask the class what patterns they see on the table. Draw out, if need be, the fact the OUT numbers are increasing by 3. Write this on the table:

IN	OUT	Difference
0	1	
1	4	3
2	7	3
3	10	3
4	13	3

Put a graph paper transparency on the overhead.

"When the IN value is 0, what is the OUT value?" Put this ordered pair on the graph.

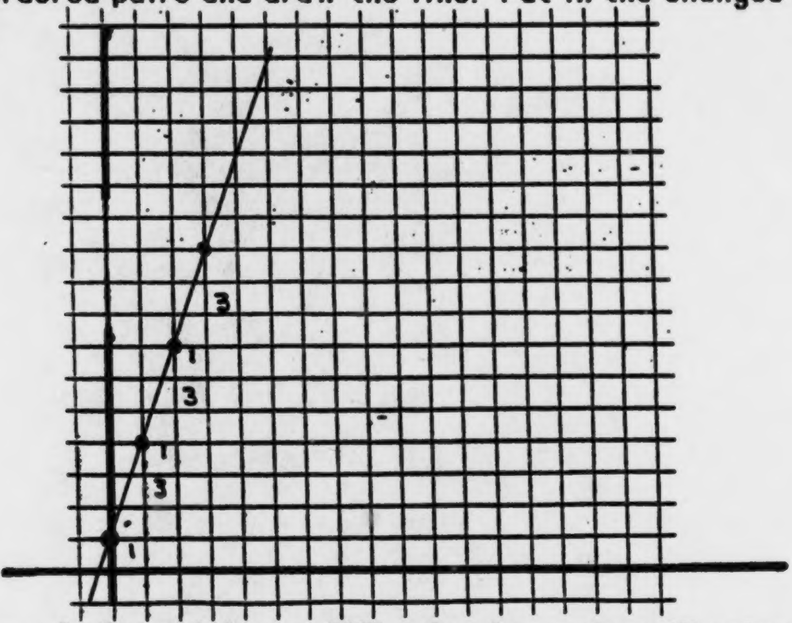


"The IN value changes by how much each time?"

"The OUT value changes by how much each time?"

"The OUT is changing three times as fast."

Graph the ordered pairs and draw the line. Put in the changes and label as shown.



"What is the rule for getting an OUT value for a given IN value?"

OUT = 3 X IN + 1 or 3 X IN + 1 = OUT or IN X 3 + 1 = OUT

"Why is an IN value of 0 important to know?"

Point out how it tells you where the line crosses the vertical axis. Leave the table and graph visible and use the rule:

2 x IN + 3 = OUT

The results of this should be:

IN	OUT	Difference
0	3	
1	5	2
2	7	2
3	9	2
4	11	2
5	13	2

"Notice the OUT value for IN = 0 shows where the line crosses the OUT (vertical) axis?"

"Point out this pair on the table."

"Where on the table do you find the number that will be a multiplier of IN in the rule?"

Give a third rule - 4 x IN + 3 and discuss the table and the graph. Show how 2 x IN + 3 and 4 x IN + 3 are alike (cross the OUT axis in the same place) and different (the graph of the latter is a "steeper" line.) Point out how the OUT values are increasing faster with the 4 as a multiplier. Pass out the worksheets and monitor the work.

LESSON FOUR

This lesson is to emphasize the idea of VARIABLE. Write the following open sentence on the board:

$$\square + 3 = 8$$

Point to the box. "A number can be put here to make this sentence TRUE.
Until a number is chosen, there are many possible numbers that MIGHT WORK. Write this on the board.

$$\square + \bigcirc = 12$$

"Make a table of numbers that could be put in the □ and the ○ to make the sentence TRUE.

□	○

Allow time and discuss these. "The □ doesn't always have to be filled with the same number to make this sentence true like it does in:

□ + 3 = 8

"The number VARIES depending on what is used with it as the ○ number."

"When there are several numbers that could be used to make number sentences true, the place holder for these numbers is called a VARIABLE. Some examples of variables you have seen are:

□, ○, **N**, ______

Give examples of each:

"Variables STAND IN PLACE of specific numbers in sentences. They are OPEN until a specific number is put in and then become CLOSED."

OPEN	**□ + 3 = 5**	**□ is a variable**
CLOSED	**8 - 3 = 5**	**VARIABLE has been replaced by a specific value**

LESSON FIVE

This lesson is to have students identify DEPENDENT and INDEPENDNT variables. It also should give them some idea of FREEDOM of choice. "In playing Guess My Rule, who was free to choose a number to use - you or me?"

Emphasize the fact that there is choice of a number for the IN, but that the rule then determines what the OUT HAS TO BE.

"Here words are used as VARIABLES." "IN is a variable for which you are free to substitute any number." "OUT is a variable where the number that can be used depends on the particular IN number." "We call IN the INDEPENDENT variable." "We call OUT the DEPENDENT variable because its number value can't be found until a number value is given to the IN variable."

"It isn't always clear from just a rule which is which. Consider:

$$\square = 2 \bigcirc + 1$$

If I give a number to □, it is the independent variable. Write:

$$7 = 2 \bigcirc + 1$$

Now the value of ○, the dependent variable, can be found. If I give a number to ○, it becomes the independent variable.

$$\square = 2 \times 4 + 1$$

and the value of □, the dependent variable can be found. The important thing is which one freedom of choice is assigned to."

LEVEL FIVE

RATIO AND PROPORTION

LESSON ONE: Reviewing Distributivity

Introduction: Students will have had an introduction to the distribution of multiplication over addition and subtraction. This is to review that experience.

On the overhead, put the following:
O O O O O O
O O O O O O
O O O O O O

"What are the dimensions of this array?"

"How can we write the 6 another way?" Do what is suggested.

Example 1:

"6 = 4 + 2"

Arrange the chips as:
O O O O   O O
O O O O   O O
O O O O   O O

Write: 3 (6) = 3 (4 + 2) = 3 (4) + 3 (2)

Example 2:

"6 = 7 - 1"

Arrange the chips as:

O O O O O | O
O O O O O | O
O O O O O | O

to show 3 (7)

Then cover the last column to show 3 (1) so

3 (6) = 3 (7 - 1) = 3 (7) - 3(1)

Write these and ask the students to tell what it is expressed as the sum of products:

4 (5 + 3) = 4 (5) + 4 (3) = 20 + 12
5 (8 + 2) = 5 (8) + 5 (2) = 40 + 10
3 (9 + 1) = 3 (9) + 3 (1) = 27 + 3
6 (2 + 7) = 6 (2) + 6 (7) = 12 + 42

Write these.

"Express these as the DIFFERENCE OF two products."

2 (8 - 3) = 2 (8) - 2 (3) = 16 - 6 = 10
3 (9 - 4) = 3 (9) - 3 (4) = 27 - 12 = 15
5 (10 - 3) = 5 (10) - 5 (3) = 50 - 15 = 35
6 (3 - 1) = 6 (3) - 6 (1) = 18 - 6 = 12

Have students complete the worksheet in pairs. Have blocks or chips available for those who need to model the problems.

LESSON TWO: Review

Introduction: Students will have had an introduction to this using UNIFIX, COLORED CHIPS and CUISENAIRE RODS. In these lessons, this will be developed further to problem solving using ratio and proportion.

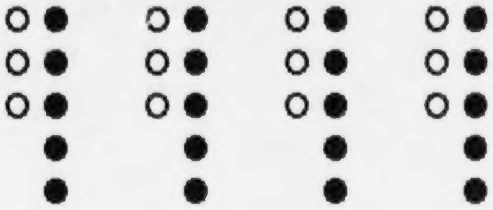
"In a jar, there are 3 red marbles for every 5 black marbles. If the jar has 12 red marbles, how many black marbles are there?"

"We'll use red and black chips to represent the marbles." Place these on the overhead:

○ ●
○ ●
○ ●
●
●

"How many of these collections would we take to have twelve red marbles?" "OK, since 12 ÷ 3 = 4."

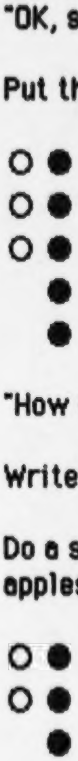
Put these on the overhead:



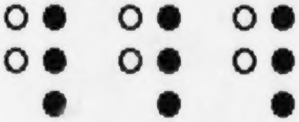
"How many black marbles are there?" "This is 4 x what number?"

Write: 4 x (3:5) = 12:20

Do a second example: "There are 2 apples for every 3 oranges. A boy has 6 apples. How many oranges are in the bag?"



"How many of these collections to get 6 apples?"



"There are 9 oranges in the bag." Write: 3 (2:3) = 6:9

Assign the worksheet to pairs of students to work on. Give them several chips of two colors to use if necessary.

LESSON THREE: The Meaning of PER and Units

Background: In mathematics as applied in everyday activity, units are often used as the basis. For example, 50¢ per candy bar, $6.00 per pizza, etc. Students should understand this idea of PER meaning "for each."

Introduction: Use a grocery ad made into a transparency. A transparency master of one is included. Translate each price into a PER statement. The pounds, cans, dozens, etc. involved are, in a way, a measurement resulting in a label. Have the students generate as many of the following kinds of statements from the ad as possible:

"peas: 69¢ per 12 oz. can."

Then take each statement generated in this way and show how they are written as:

69¢/can or $\frac{\text{69¢}}{\text{can}}$

The / symbolism or ___ is used to show the idea of rate or per.

Ask the students what other rates they can think of. The most likely is miles per hour for speed. Write this as miles/hour or $\frac{\text{miles}}{\text{hour}}$.

Consider the following: "Eight apples are bought at 40¢ per apple." Rate is $\frac{\text{40¢}}{\text{apple}}$

Total is 8 apples x $\frac{\text{40¢}}{\text{apple}}$ = ?

Most students will see the result must be money. What must happen to the apple labels? These are "Cancelled" just like numbers:

$\frac{4}{4} = 1$ $\qquad$ $\frac{\text{apples}}{\text{apples}} = 1$

Have the students work in pairs on the worksheets provided.

LESSON FOUR: Using PER

Background: One way of applying ratio (or rate) and proportion is to make use of the UNITARY RATE, or the amount of a quantity per unit. Then one can multiply the unitary rate by any number of those units.

Introduction: Give this problem:

"Joyce buys a package of 10 audio tapes for $12.99. Sam buys the same kind in a package of 3 for $3.99. Who gets the better buy? Why?

"What is the cost of ONE tape for Joyce?"
"What is the cost of ONE tape for Sam?"
"The UNITARY RATE or cost of ONE allows you to compare."

Give another problem:

"Tom's father drives the 30 miles to Forest City in 45 minutes. At the same speed how far could he drive in one hour?"

"What quantities are in the rate?" (miles and minutes)
"How do we show this?" $\left(\dfrac{30 \text{ miles}}{45 \text{ minutes}}\right)$
"What is the rate in miles in ONE minute?" $\left(\dfrac{2}{3} \text{ miles}\right)$
"Now we can multiply these miles in ONE minute by any number of minutes to get total miles."

"How many minutes in one hour?" "We multiply 60 (2/3 miles) = 40 miles - is the distance Tom's father could travel in (ONE) hour."

Have pairs of students work on the problems given. Remind them to find the UNITARY RATE first.

LESSON FIVE: Setting up Proportions

Introduction: In using ratio and proportion to solve problems, students should:

1. Be aware of the unitary rate
2. Label all numbers with "units" of some kind
3. Cancel units to see what units the answer is in.

Completely analyze and discuss the following problem:

"A store finds one bad egg in every 4 dozen eggs. In a crate of 48 dozen how many bad eggs should the store expect to find?"

"The given comparison is $\left(\dfrac{1 \text{ bad egg}}{4 \text{ dozen eggs}}\right)$"

"What part of this is multiplied to give what is in the problem?"

"4 dozen is multiplied by what to get 48 dozen?"

"We multiply the other part by 12 as well": $\left(\frac{1 \text{ bad egg}}{4 \text{ dozen}}\right) 12 = \frac{12 \text{ bad eggs}}{48 \text{ dozen}}$

Consider a second problem: "In one minute, Terri drank 4 oz. of soda and George drank 6 oz. of soda. How much would each drink in 3 minutes?"

Terri	George
$\frac{4 \text{ oz.}}{\text{min.}} \times \frac{3 \text{ min.}}{1} = 12 \text{ oz.}$	$\frac{6 \text{ oz.}}{\text{min.}} \times \frac{3 \text{ min.}}{1} = 18 \text{ oz.}$

Emphasize the cancelling of the units to see how the units appear in the answer.

ARITHMETIC PROBLEM SOLVING AT LEVEL FIVE

By the end of this experience, children should be able to solve any type of the arithmetic operation problems.

Consider these factors when working with children with arithmetic problem solving:

Some students may still need some concrete materials to represent the thing given orally or in written situations........

Students should write number sentences that model the conditions of the problem given......

Students should be given opportunities to see a variety of problem solving being used
.............guessing and checking the guess
.............drawing pictures
.............making diagrams
.............making tables or graphs
.............estimating
.............writing number sentences to model

Non-numeric problem solving situations should also be presented using Pattern Blocks, Tangrams and other right hemisphere related materials.*

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*See Arithmetic Teacher
March, 1986 "Verbal Addition and Subtraction Problems"
April, 1986 "Verbal Multiplication and Division Problems"

Verbal Addition and Subtraction Problems: Some Difficulties and Some Solutions

By **Charles S. Thompson** *and* **A. Dean Hendrickson**

Many of the difficulties that children have in solving verbal (story) problems involving addition and subtraction arise because of their limited understanding of the arithmetic operations that are involved. They don't know when to use addition or subtraction because they lack specific knowledge regarding the various situations that give rise to these operations. Often, children are taught addition only as "putting together" and subtraction only as "taking away," but many other settings involve addition and subtraction operations. Children need to receive specific instruction in different contexts if they are to become good solvers of verbal addition and subtraction problems. This article describes the contexts and then explains a successful sequence of activities that teach verbal problems.

Table 1
Change Problems

Problem title	Sample problem	Characteristics
Change 1	Bill has two pencils. Jean gives him three pencils. How many pencils does Bill have then?	Increase, initial set and change set known, question about final set
Change 2	Bill has five pencils. He gives three to Jean. How many pencils does he have left?	Decrease, initial set and change set known, question about final set
Change 3	Bill has two pencils. Jean gives him some more. Now he has five. How many did Jean give him?	Increase, initial set and final set known, question about change set
Change 4	Bill has five pencils. He gives some to Jean. Now he has two. How many did he give to Jean?	Decrease, initial set and final set known, question about change set
Change 5	Bill has some pencils. Jean gave him two more. Now he has five. How many did he begin with?	Increase, change set and final set known, question about initial set
Change 6	Bill has some pencils. He gave three to Jean. Now he has two. How many did he begin with?	Decrease, change set and final set known, question about initial set

Categories of Verbal Problems

In elementary school mathematics, three categories of verbal problems suggest addition and subtraction operations. These categories—Change, Combine, and Compare—are described by Nesher (1981). Various types of problem situations exist within each category.

Let's look first at the Change category. Change problems involve increasing or decreasing an initial set to create a final set. One sample Change problem is a familiar "putting together" situation (fig. 1).

> Bert has two books. On his birthday he gets three new books. How many books does Bert have then?

All Change problems have three quantities: an initial set, a change set, and a final set. In the problem given, the *initial set* is two books, the *change set* is three books, and the *final set* is unknown. The unknown quantity in Change problems can be any one of the three sets, yielding three kinds of problems. Furthermore, the change can be either an increase or a decrease, thus yielding two problems for each of the three kinds, for a total of six types of Change problems. These problems are described and characterized in table 1.

The second category of problems is called Combine, or part-part-whole. Combine problems describe an existing, static condition involving a set and its several component subsets. A major difference between Change and Combine problems is that no action is involved in Combine problems. A sample problem is as follows:

> Consuelo has five buttons. Three are round and the rest are square. How many are square?

See figure 2.

A typical Combine problem has three related quantities—one subset, the other subset, and the whole set. These yield only two types of problems. In our example, the whole set and one subset are known. In the

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Fig. 1 Change problems involve increasing or decreasing an initial set to create a final set.

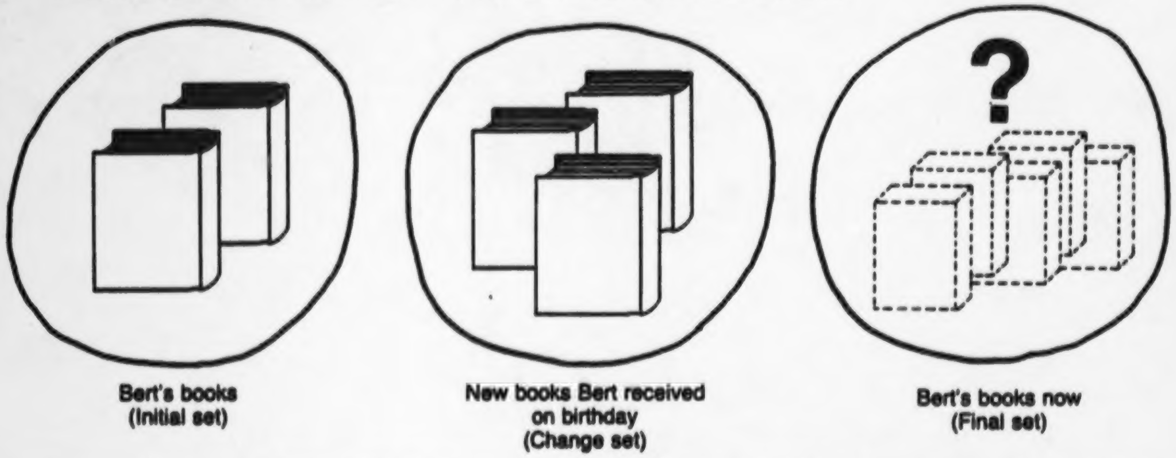

?
Bert's books
(Initial set)
New books Bert received
on birthday
(Change set)
Bert's books now
(Final set)

Fig. 2 "Combine" problems describe an existing condition involving a set and its several component subsets.

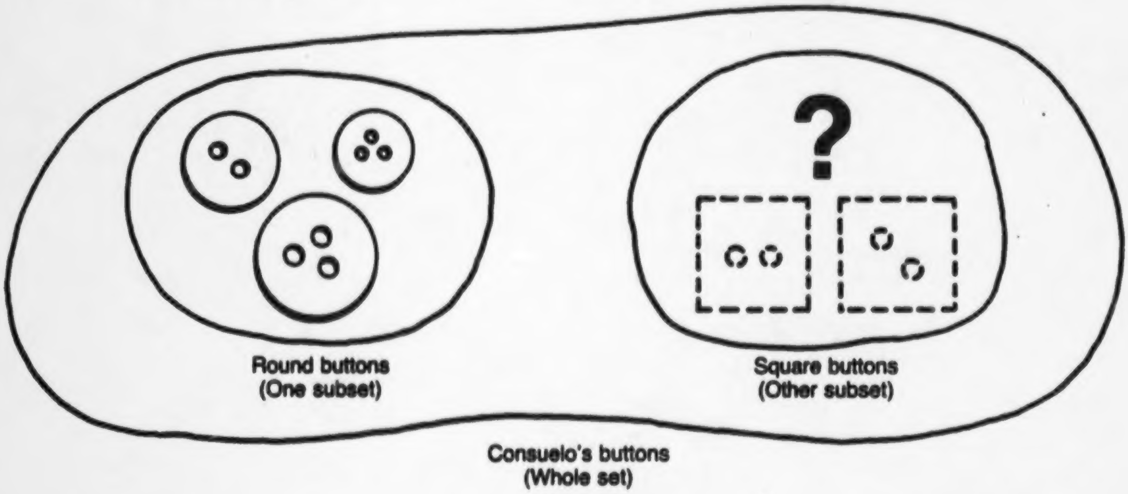
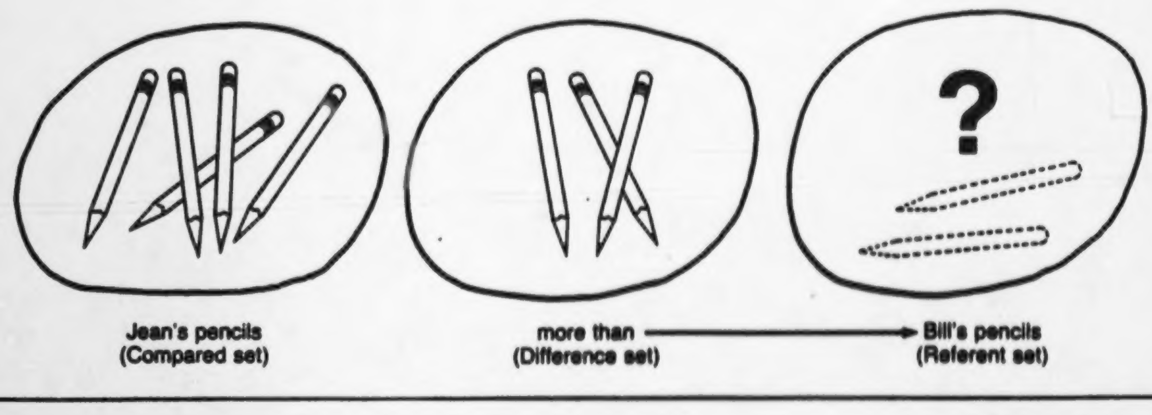

?
Round buttons
(One subset)
Square buttons
(Other subset)
Consuelo's buttons
(Whole set)

Fig. 3 Compare problems involve a comparison of two existing sets.


?
Jean's pencils
(Compared set)
more than
(Difference set)
Bill's pencils
(Referent set)

other type of problem, both subsets are known and the whole set is unknown. Table 2 summarizes these Combine problems.

Other Combine problems involve more than two subsets and the whole set. These problems typically involve a two-step process and are not discussed here.

The third category of problems is called Compare. Compare problems, which involve a comparison of two existing sets, are probably the most ignored type of problem in school curricula. Yet many children's experiences involve comparisons. Here is a sample problem:

> Jean has five pencils. She has three more pencils than Bill. How many pencils does Bill have?

See figure 3.

Each Compare problem has three expressed quantities—a referent set, a compared set, and a difference set. The referent set is the set to which the comparative description refers. In the sample problem, Bill's pencils compose the referent set, since Jean "has three *more* pencils *than Bill*." The compared set is the set being compared to the referent set. In the sample problem, Jean's set of five pencils (the compared set) is compared to Bill's set (the referent set). The difference set is the difference between the referent set and the compared set.

There are six types of Compare problems. The unknown quantity can be the referent set, the compared set, or the difference set. For each of these three possibilities, the comparison can be stated in two ways: (1) the (larger) compared set is *more than* the (smaller) referent set, or (2) the (smaller) compared set is *less than* or *fewer than* the (larger) referent set. Table 3 summarizes and gives examples of the six types of Compare problems.

Table 2
Combine Problems

Problem title	Sample problem	Characteristics
Combine 1	Bill has three red pencils and two green pencils. How many pencils does Bill have all together?	Two subsets are known, question about whole set
Combine 2	Bill has five pencils. Three are red and the rest are green. How many are green?	Whole set and one subset are known, question about other subset

Table 3
Compare Problems

Problem title	Sample problem	Characteristics
Compare 1	Bill has two pencils. Jean has five. How many more does Jean have than Bill?	Comparison stated in terms of "*more*," referent set and compared set known, question about difference set
Compare 2	Bill has two pencils. Jean has five. How many fewer pencils does Bill have than Jean?	Comparison stated in terms of *less (fewer)*, referent set and compared set known, question about difference set
Compare 3	Bill has two pencils. Jean has three more than Bill. How many pencils does Jean have?	Comparison stated in terms of *more*, referent set and difference set known, question about compared set
Compare 4	Jean has five pencils. Bill has three fewer pencils than Jean. How many pencils does Bill have?	Comparison stated in terms of *less (fewer)*, referent set and difference set known, question about compared set
Compare 5	Jean has five pencils. She has three more pencils than Bill. How many pencils does Bill have?	Comparison stated in terms of *more*, compared set and difference set known, question about referent set
Compare 6	Jean has two pencils. She has three fewer pencils than Bill. How many pencils does Bill have?	Comparison stated in terms of *less (fewer)*, compared set and difference set known, question about referent set

Relative Difficulties of Verbal Problems

Examination of the various types of problems and observations of children solving these problems lead to the conclusion that some types of problems are more difficult to solve than others. In general, it appears that the inherent structure of the problem is the crucial factor in determining its difficulty. For example, Combine-1 problems are structurally straightforward (table 2).

> Combine 1. Bill has three red pencils and two green pencils. How many pencils does Bill have all together?

The two subsets are given. Children can count those subsets separately. Then, they must simply recount the entire collection of objects to determine the solution to the problem. Or, depending on instruction they have received, they might use "all" or "all together" to transform it to a Change problem.

Combine-2 problems, by comparison, are not straightforward. The sets to be considered are not separate from one another.

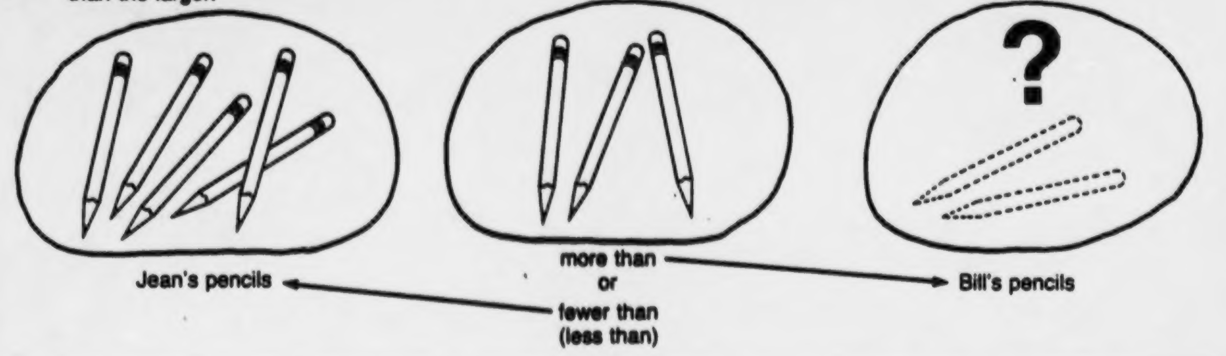
> Combine 2. Bill has five pencils. Three are red and the rest are green. How many are green?

The children must have a well-developed part-whole understanding. The whole set and one subset are given. To solve this kind of problem, children must know that the given subset is contained within the whole set mentally or physically to separate that subset from the whole set and then count the other subset. This problem can be transformed correctly into a Change-2 problem by many children. Other children transform it incorrectly into a comparison of the two subsets.

Another major factor affecting the difficulty of a problem is its semantics. How the relationships between the sets are expressed determines, to some extent, which cognitive structures must be used by the child to solve the problem. For example, study the following Compare-4 and Compare-5 problems:

Compare 4. Jean has five pencils. Bill

Fig. 4 The child must understand that the larger set is three more than the smaller set and the smaller set is three less than the larger.



has three pencils *fewer than* Jean. How many pencils does Bill have?

Compare 5. Jean has five pencils. She has three *more* pencils *than* Bill. How many pencils does Bill have?

See figure 4.

In each problem the larger set, of the two being compared, and the difference set are given. The child is to determine the smaller set. In the Compare-4 problem the expression used to relate the larger and smaller sets is "The smaller set is three pencils *fewer than* the larger (known) set." To solve this problem, the child might simply create what is described, by removing three pencils from the larger set to create the smaller set of two objects. This behavior transforms the problem into a Change-2 problem. In the Compare-5 problem, however, the statement used to relate the larger and smaller sets is, in effect, "The larger set is three pencils *more than* the smaller (but unknown) set." In this problem the child must use a different cognitive structure to determine what to do. Three pencils cannot be added to the smaller set, since its quantity is not known. The child must understand that if the larger set is three *more than* the smaller set, then the smaller set is three *fewer than* the larger. The child must have a well-developed cognitive structure called *reversibility*. The child must understand that the statement "*x* is *a* more than *y*" is equivalent to "*y* is *a* less than *x*." Only then will the child know that removing objects from the larger set will create the "more than" relationship expressed in the verbal problem. This same reversibility enables some children to transform Combine-2 problems into Change-2 problems.

Another factor affecting the difficulty of Compare problems is that in Compare-3, 4, 5, and 6 problems, the difference set must be mentally constructed by the child. It is not actually part of the compared set or the referent set. Furthermore, after the difference set is mentally constructed, the child must mentally add it to, or subtract it from, one given set to determine the unknown set.

Another difficulty is the varying use of the expressions *more than, less than*, and *fewer than*. The phrase *fewer than* is common in these fourteen types of problems, since discrete, countable sets are involved. *Fewer than* suggests counting strategies more readily than does *less than*. However, *more than* is used to express relationships between either countable or noncountable quantities. Further, the word *more* is often used in Change problems in another way, as in "John gave Frank four more."

The relative difficulties of all fourteen types of verbal problems have not yet been fully determined. But informal observations of children solving these problems, careful analysis of the problems' structures and semantics (Nesher et al. 1982), and analysis of research results (Carpenter and Moser 1981; Nesher 1981; Riley 1981; Steffe 1971; Tamburino 1981) provide preliminary information about the difficulty of problems. Currently available information indicates four levels of difficulty:

Easiest: 1. Change 1 & 2, Combine 1
2. Change 3 & 4, Compare 1 & 2
3. Combine 2, Change 5 & 6, Compare 3 & 4
Hardest: 4. Compare 5 & 6

Instructional Procedures

We have been working in a conceptually oriented, materials-based elementary mathematics program. The children in first, second, and third grades have received instruction in solving verbal problems of the fourteen types that have been described. The following general instructional sequence has been followed over a period of weeks:

1. Problem situations are presented orally to children. The children use countable materials that can be grouped, linked, and separated to aid them in solving problems. Their answers are expressed *orally*.

2. Children use countable objects to explore combinations of numbers that make larger numbers. For example, they separate five counters into two subsets in different ways and describe the results orally, such as "three and two" or "one and four."

3. Children use prepared numeral cards (0-9), and cards with the "+," "−," "=," and "☐," in conjunction with activities similar to those previously described in step 2. They con-

struct number phrases and sentences with the prepared sign cards to represent the objects being used. This task helps them to connect the signs to the concepts involved. For example, if a child uses five counters and covers two of them, then a partner can create the open sentence 3 + □ = 5 then insert a "2 card" to complete the open sentence.

4. The problem situations are presented orally to children as in step 1. They use countable objects to solve the problems and now use the *prepared cards to construct number sentences* to represent the objects used and the conditions of the problem. For example, consider the following problem:

Change 1. Bill has two pencils. Jean gives him three pencils. How many pencils does Bill have now?

To solve this problem, children frequently make separate links of cubes to represent the two sets, join the two links, and arrange cards as shown:

$\boxed{2}\ \boxed{+}\ \boxed{3}\ \boxed{=}\ \boxed{5}$

5. Children use countable materials to solve orally presented problems and then *write number sentences* to indicate how they interpreted the problems. In particular, children circle their answers in the number sentences. In many problem situations several possible number sentences can be written. Consider this problem:

Compare 1. Jean has five pencils. Bill has two pencils. How many more pencils does Jean have than Bill?

Some children will interpret this as an addition problem and write 2 + ③ = 5. Others will interpret it as subtraction and write 5 − 2 = ③. Both interpretations are correct.

6. Open sentences in written form are given to children, who use countable materials to solve them.

7. Materials are not used, and children solve written verbal problems mentally while writing the corresponding number sentences.

8. Children solve open sentences (not directly tied to verbal problems) in written form without the use of countable materials.

From a broad perspective, the sequence has used the following steps: (1) develop concepts using materials, (2) connect signs to the concepts, (3) construct symbolic forms (number sentences) using prepared symbols, (4) write symbolic forms, and (5) interpret prepared symbolic forms. This sequence has resulted in students being able to interpret these problems and translate them into number-sentence models.

In conjunction with these activities, children participate in numerous counting exercises. They learn to count on from any given number and to count back from any given number. Counting on is useful in many problems, particularly in part-whole situations, in which one subset and the whole set are known, and in compare situations, where equalizing of the two sets is the strategy to be used. Counting back is also used frequently, especially in Change problems. For example, in Change-2 problems the children often count back from the larger (initial) set to create the smaller (final) set.

Instructional Results So Far

The instructional sequence described seems to be effective in enabling children in the primary grades to solve verbal problems. Of crucial importance seem to be the use of countable materials, the use of the prepared numeral and sign cards, and the practice of circling answers when writing number sentences.

Using the countable materials enables the children to create or model the conditions presented in the problems. The children can then determine which sets to count, compare, separate, or join to solve the problems. The use of the prepared cards allows the children quickly to attach numerals to the quantities represented and to construct the corresponding number sentences. We have found that children who have not used numeral cards experience greater difficulty in writing number sentences corresponding to a verbal problem. The practice of having children circle answers when writing number sentences helps teachers understand how the children are thinking about the verbal problems. Indeed, for many of the types of problems, either an addition or a subtraction number sentence is appropriate. These practices also help teachers to recognize when children are successfully using the class-inclusion relation, reversibility of both actions and relations, and equalization of two sets.

In summary, we have learned that children can become good solvers of verbal problems. What they need is an instructional program that proceeds from the concrete to the symbolic *and* the opportunity to encounter the various problem situations that occur in real life.

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Verbal Multiplication and Division Problems: Some Difficulties and Some Solutions

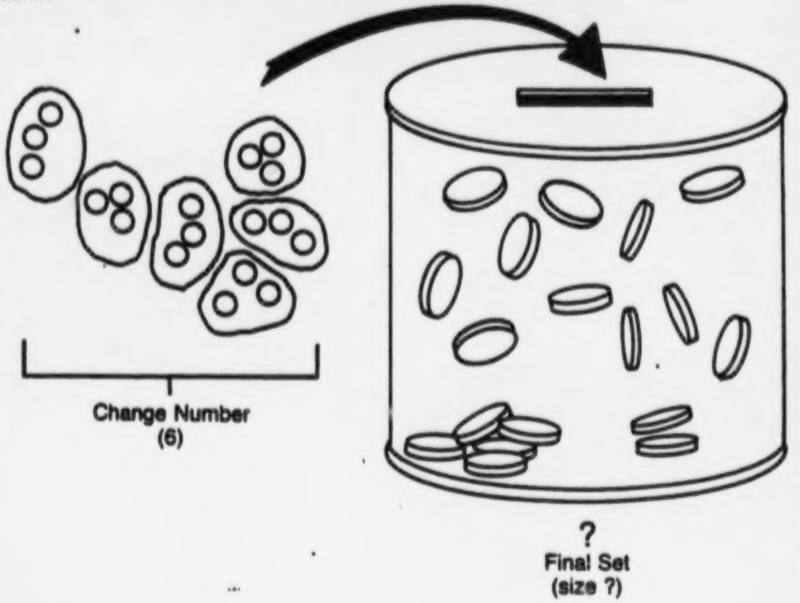
By **A. Dean Hendrickson**

Verbal problems that involve multiplication and division are difficult for children to solve. Many of these difficulties arise because of their limited understanding of these arithmetic operations. Their experience with the different kinds of situations that call for these operations is also limited. At the same time, these problems cannot be categorized easily because the situations that require these operations are varied. Nonetheless, multiplication is often taught only as "repeated addition" and division only as "repeated subtraction." Children must have specific instruction in *all* the situations that require multiplication and division as arithmetic operations if they are to apply them successfully to verbal problems.

Change Problems

Extensions of the "change problems" for addition and subtraction can lead to multiplication and division. In this particular kind of problem we have an initial set, a change number, and a final set. Given an initial set of small size and a change number that describes how many of this size set are joined, we find the size of the larger final set by multiplication. These problems are *change 1*, or repeated addition, problems. Here is an example (fig. 1):

Fig. 1 An example of a *change 1* or "repeated addition" problem: John put 6 handfuls of 3 tokens each into a bank. How many tokens did John put in the bank?



John put 6 handfuls of 3 tokens each into a bank. How many tokens did John put in the bank?

Change 2 problems result when a large initial set is given along with the size of a smaller final set, and a change number needs to be found that describes *how many* sets of that size can be made from the initial set. This problem represents the *measurement*, or repeated-subtraction, interpretation of division. Here is an example (fig. 2):

Susie has 24 cookies. She gives 3 cookies to each of the children on the playground. How many children are on the playground?

A child who can reverse the "putting together" transformation can relate a measurement interpretation of the division of countable materials to the repeated-addition kind of multiplication. In some ways the division is easier, since the child must retain only the final set size and count the number of sets that can be made. The count is constructed in the process and the size of the initial set is not important, since the count stops whenever the process runs out of objects. In repeated addition, both the count num-

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ber and the size of the initial set must be retained mentally along with the result at the end of each successive joining.

Change 3 problems involve a large initial set and a known change number; the size of the final, *equal* sets that can be made from the initial set must be found. This is the *partition* interpretation of division. An example follows (fig. 3):

> Susie has 24 cookies. She gives an equal number to each of her 4 friends. How many cookies does each friend get?

Change 2, or measurement division, is easier, since only the size of the set being formed repeatedly must be retained and a count of these sets kept as they are made. Change 3, or partition division, requires a strategy to assure the equality of the sets being made and hence is more difficult.

Comparison Problems

Questions involving "less than" or "more than" lead to addition and subtraction problems. These problems involve a comparison set, a difference set, and a referent set. When we compare two sets and the comparison involves questions of "how many times as many" or "what part of," we use multiplication and division. Such problems involve a comparison set, a referent set, and a correspondence other than a one-to-one correspondence between these sets. In figure 4, if the question is asked, "*A* has how many times as many as *B*?" then *A* is the comparison set, *B* is the referent set, and the correspondence of *A* to *B* is sought.

Compare 1 problems result when the referent set and a many-to-one correspondence are given and students are asked to find the comparison set. The following is an example (fig. 5):

> Iris has 3 times as many nickels as dimes. She has 4 dimes. How many nickels does she have?

Multiplication is used to find the answer: 3 × 4 = 12.

Compare 2 problems occur when the comparison and a many-to-one

Fig. 2 An example of a *change 2* problem, measurement or repeated-subtraction interpretation of division: Susie has 24 cookies. She gives 3 cookies to each of the children on the playground. How many children are on the playground?

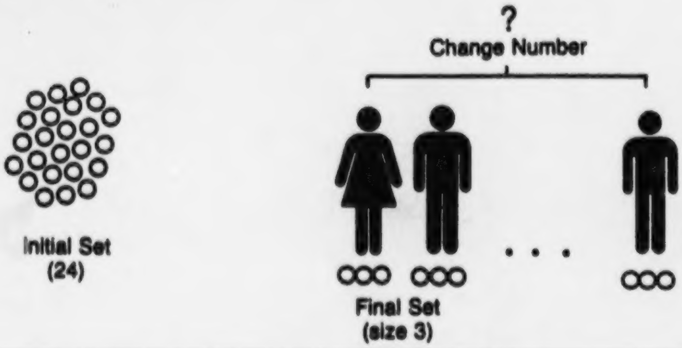


Fig. 3 An example of a *change 3* problem, a partition interpretation of division: Susie has 24 cookies. She gives them in equal numbers to her four friends. How many cookies does each friend get?

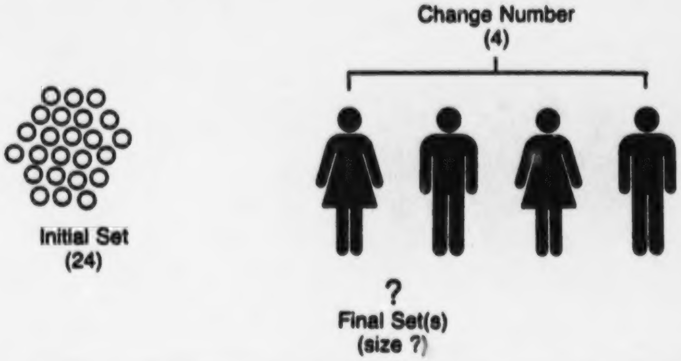


Fig. 4 A comparison problem: Find the correspondence of *A* to *B*. *A* has how many times as many as *B*?

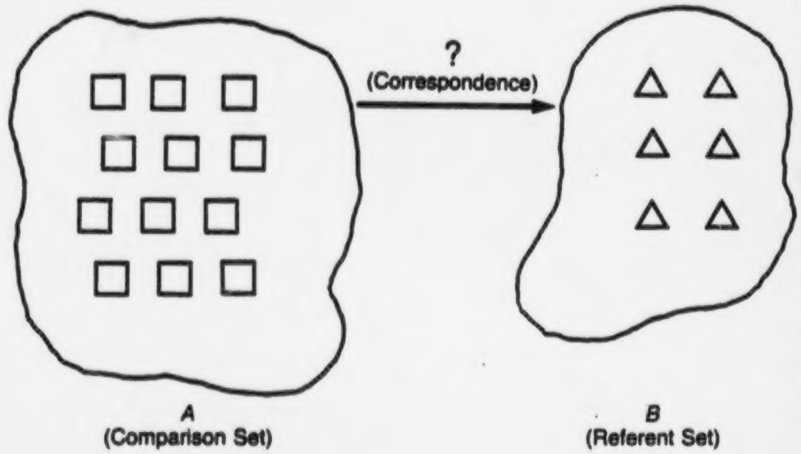


Fig. 5 A *compare 1* problem: Iris has 3 times as many nickels as dimes. She has 4 dimes. How many nickels does she have?

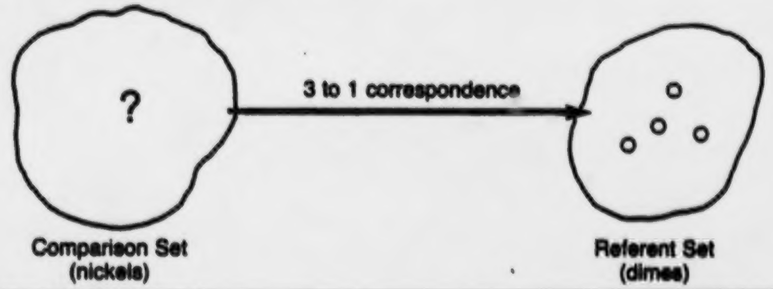

?
3 to 1 correspondence
Comparison Set
(nickels)
Referent Set
(dimes)

Fig. 6 A *compare 2* problem: Iris has 15 nickels. She has 3 times as many nickels as dimes. How many dimes does Iris have?

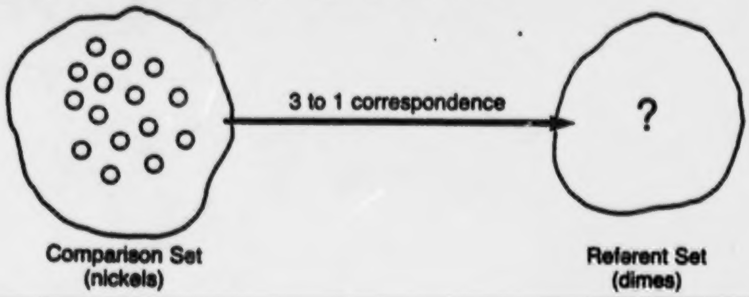

3 to 1 correspondence
?
Comparison Set
(nickels)
Referent Set
(dimes)

Fig. 7 A *compare 3* problem: Frank has 24 nickels and 8 dimes. He has how many times as many nickels as dimes?

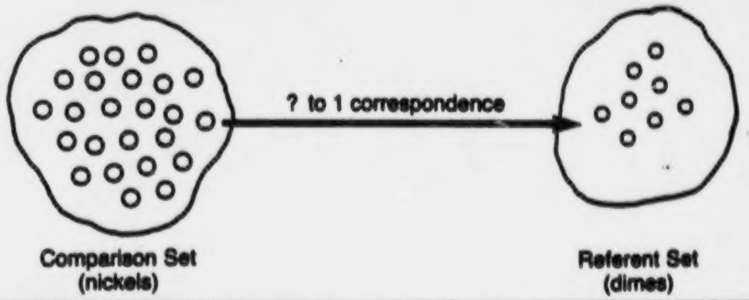
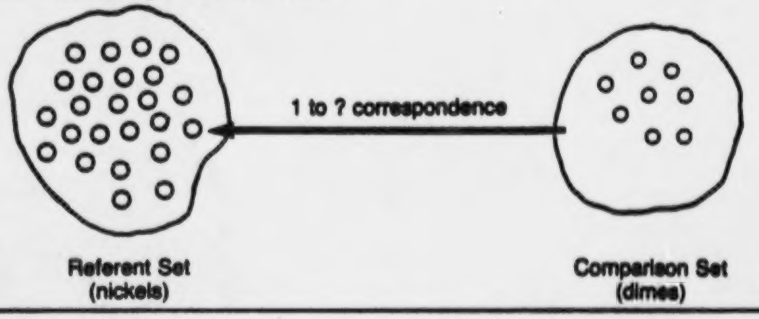

? to 1 correspondence
Comparison Set
(nickels)
Referent Set
(dimes)

Fig. 8 A *compare 4* problem: Frank has 24 nickels and 8 dimes. He has what fraction as many dimes as nickels?


1 to ? correspondence
Referent Set
(nickels)
Comparison Set
(dimes)

correspondence are given and the referent set must be found. Here is an example (fig. 6):

> Iris has 15 nickels. She has 3 times as many nickels as dimes. How many dimes does Iris have?

Division is used to find the answer: 15 ÷ 3 = 5.

Compare 3 problems result when the comparison set and referent set are known and a many-to-one correspondence must be found (fig. 7):

> Frank has 24 nickels and 8 dimes. He has how many times as many nickels as dimes?

Division is used to find the answer: 24 ÷ 8 = 3.

Compare 4 problems occur when a comparison set and a referent set are given and a one-to-many correspondence is sought. In this case, the comparison set is the smaller of the two. Here is an example (fig. 8):

> Frank has 24 nickels and 8 dimes. He has what fraction as many dimes as nickels? (or, Frank's dimes are what fractional part of his nickels?)

The result is division of a smaller by a larger number or formation of a rational number, usually expressed as a fraction: 8 ÷ 24 = 1/3.

This kind of question puts a child's concept of *fraction* being equal parts of a whole into conflict with this ratio situation. What other language can be used to ask for this correspondence? Because of the difficulty of finding suitable language, questions related to finding this correspondence are seldom found in textbooks.

Compare 5 problems arise when the comparison set and the referent set are given and a many-to-many correspondence is sought (fig. 9):

> There are 12 girls and 16 boys in the room.. How many times as many boys are there as girls?

One divides to find the answer (16 ÷ 12 = 4/3). Here again a fraction tells how many times as much, although a ratio correspondence is made in the thinking.

Compare 6 problems occur when the comparison set is smaller than the referent set and the correspondence is sought (fig. 10):

> There are 12 girls and 16 boys in a room. The number of girls is what part of the number of boys?

The result is found by division again, 12 ÷ 16 = 3/4, and the same conflict between ratio and fraction results.

Compare 7 problems result when the larger comparison set and the many-to-many correspondence are given and the size of the smaller referent set is sought (fig. 11):

> There are 16 boys in a class. There are 4/3 as many boys as girls. How many girls are there?

The answer is found by dividing: 16 ÷ 4/3 = 12.

Compare 8 problems arise when the smaller referent set is given along with a many-to-many correspondence. The size of the larger comparison set is sought (fig. 12):

> There are 12 girls in the room. The number of boys is 4/3 the number of girls. How many boys are in the room?

The answer is found by multiplying: 4/3 × 12 = 16.

The compare problems that involve many-to-many correspondences are difficult, since they bring into conflict the child's recognition of a fraction as comparing a given number of equal parts to the whole and the idea of ratio as a correspondence. The use of the same symbolism for both fractions and rational numbers compounds this difficulty.

Thinking in ratios, equating ratios, and applying ratios to situations involve formal operational thought. Very few elementary children are capable of this kind of reasoning. In fact, few eighth and ninth graders can think through the Mr. Tall–Mr. Short problem:

	Mr. Tall	Mr. Short
Measured in match sticks	9	6
Measured in paper clips	12	?

Fig. 9 A *compare 5* problem: There are 12 girls and 16 boys in the room. How many times as many boys are there as girls?

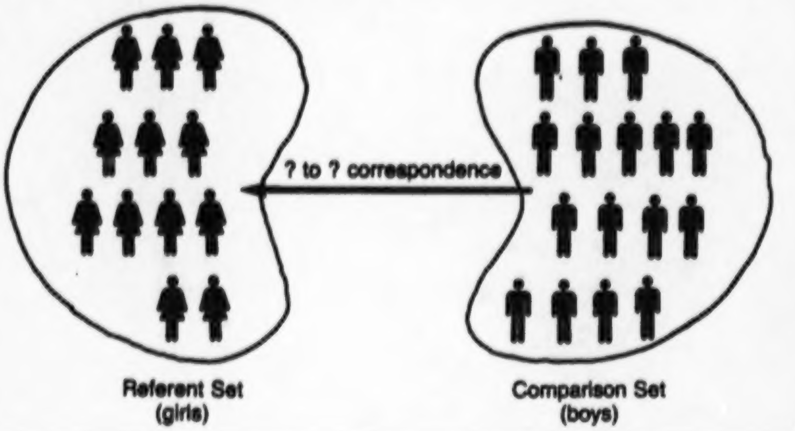


Fig. 10 A *compare 6* problem: There are 12 girls and 16 boys in a room. The girls are what part of the boys?

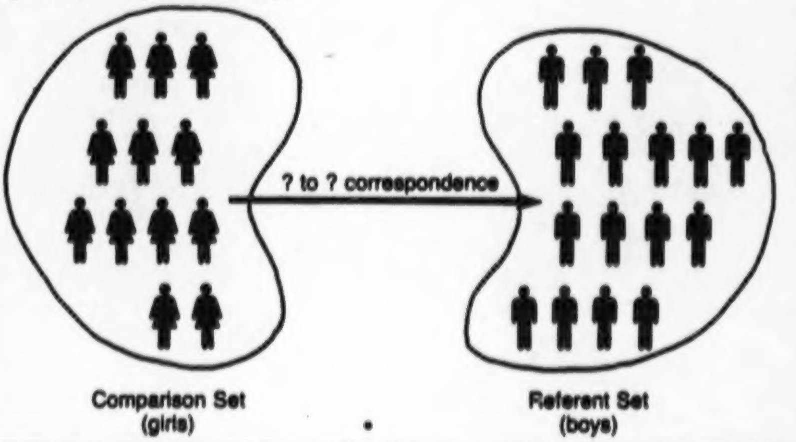


Fig. 11 A *compare 7* problem: There are 16 boys in a class. There are 4/3 as many boys as girls. How many girls are there?

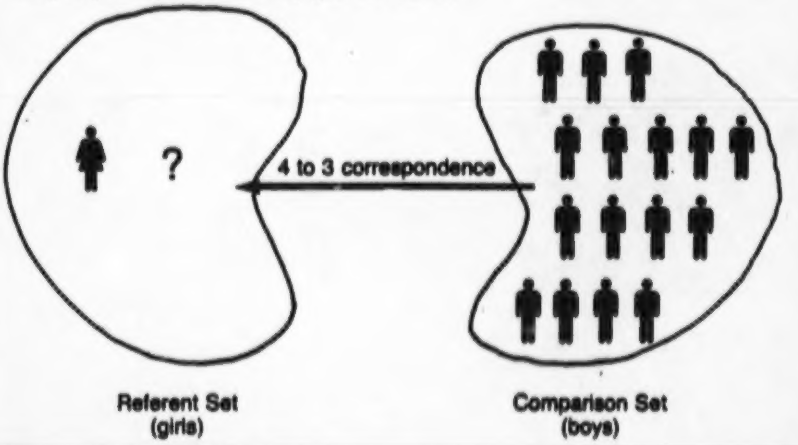
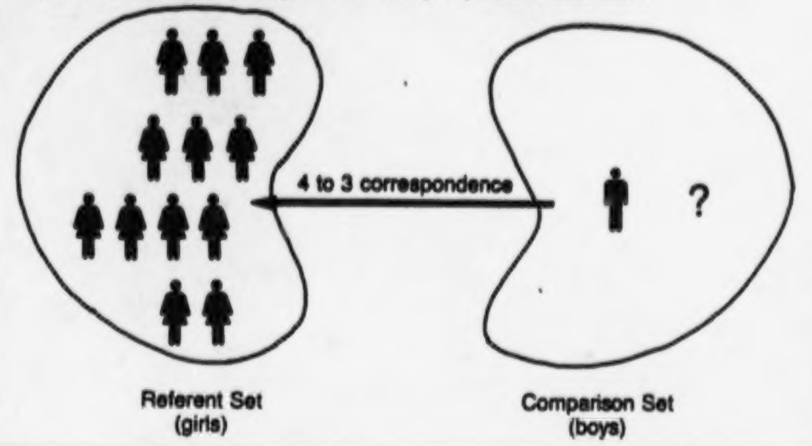


Fig. 12 A *compare 8* problem: There are 12 girls in the room. The number of boys is 4/3 the number of girls. How many boys are in the room?

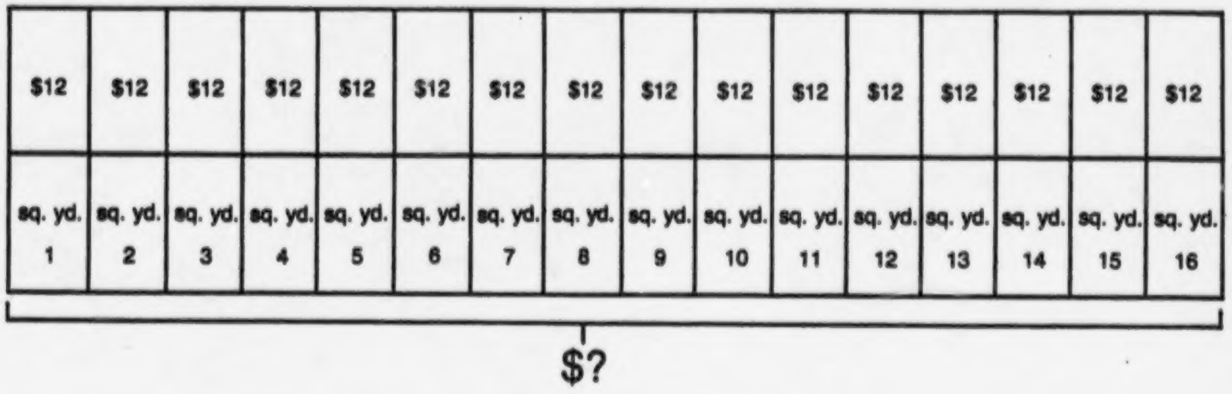

4 to 3 correspondence
?
Referent Set
(girls)
Comparison Set
(boys)

Rate Problems

The kind of proportional reasoning used in equating ratios is also involved in thinking about rate problems. These are commonly found in intermediate textbooks. A rate problem involves two variables—one independent and one dependent—and a rate of comparison between them. An example is distance (miles) = rate (miles per hour) × time (hours). Here the number of hours is the independent variable, the distance in miles (a total) is the dependent variable, and the ratio of miles to hours is the rate.

Some common rate examples are these:

Fig. 13 A *rate 1* problem: Fred pays $12.00 a square yard for outdoor carpeting. How much will 16 square yards cost?


$12
$12
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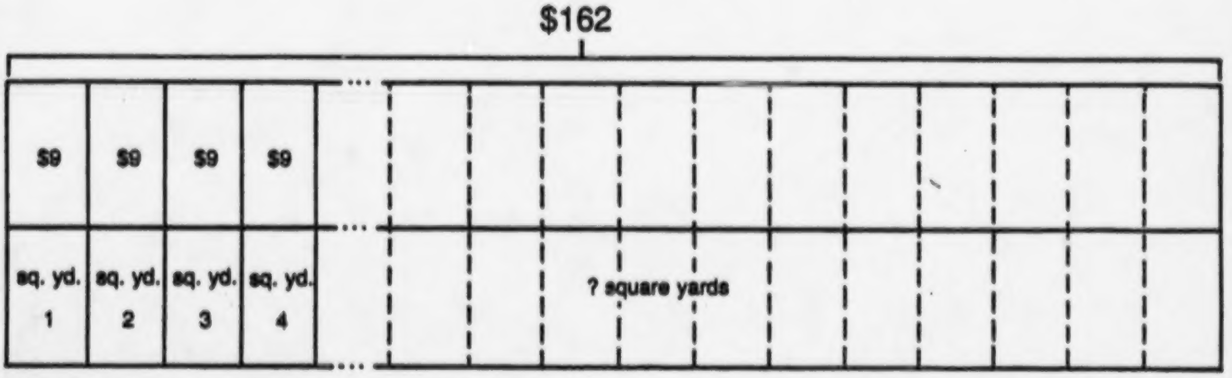
$12
sq. yd. 1
sq. yd. 2
sq. yd. 3
sq. yd. 4
sq. yd. 5
sq. yd. 6
sq. yd. 7
sq. yd. 8
sq. yd. 9
sq. yd. 10
sq. yd. 11
sq. yd. 12
sq. yd. 13
sq. yd. 14
sq. yd. 15
sq. yd. 16
$?

Fig. 14 A *rate 2* problem: Jane pays $162 for carpeting at $9 a square yard. How many square yards did she get?


$162
$9
$9
$9
$9
sq. yd. 1
sq. yd. 2
sq. yd. 3
sq. yd. 4
? square yards

- feet per second
- dollars per pound
- pounds per cubic foot
- gallons per minute
- cents per kilowatt hour
- parts per hundred

Children who are unable to think about rates and ratios will have difficulty doing these problems in any way other than substituting numbers into memorized formulas. Problems dealing with percentages are probably the best example of this difficulty.

Rate 1 problems result when the rate and the value of independent variable quantity are given (usually in units of measurement) and the value of the dependent variable, usually a total, must be found (fig. 13):

> Fred pays \$12 a square yard for outdoor carpeting. How much will 16 square yards cost him?

The resulting application,

total cost
  = cost/sq. yd. × number of sq. yd.
  = \$12/sq. yd. × 16 sq. yd. = \$192,

is the easiest of the rate situations to use.

Rate 2 problems result when the rate and the value of the dependent variable are given and the value of the independent variable is sought (fig. 14):

> Jane pays \$162 for carpeting at \$9 a square yard. How many square yards does she get?

We have

$$\$162 = \$9/\text{sq. yd.} \times \boxed{}\ \text{sq. yd.}$$

or

$$\text{sq. yd.} = \frac{\$162}{\$9/\text{sq. yd.}} = \boxed{18}.$$

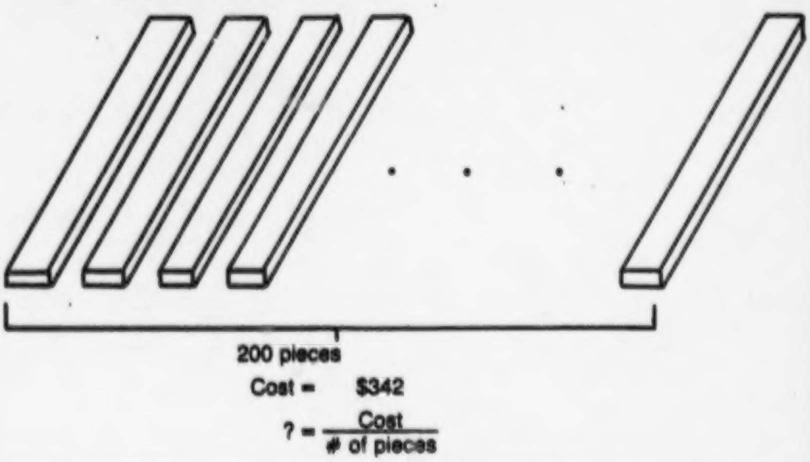
Rate 3 problems result when the values of the dependent and independent variables are given and the ratio or comparison rate is sought (fig. 15):

> Peter paid \$342 for 200 eight-foot two-by-fours. What was the cost in dollars of each two-by-four?

We have

$$\$342 = \boxed{\$}/\text{board} \times 200 \text{ boards}$$

Fig. 15 A *rate 3* problem: Peter paid \$342 for 200 eight-foot 2 x 4's. What was the cost in dollars of each 2 x 4?



or

$$\$ \text{ cost/board} = \frac{\$342}{200 \text{ boards}} = \$1.71/\text{board}$$

Selection Problems

Among the most difficult problems are those that require multiplication. These belong to a more general group of selection problems.

Selection 1 problems involve simple ordered pairs where the choice sets for each element of the ordered pair are given and the number of ordered pairs possible is sought. The pairs are ordered in the sense that one choice set is associated with one element and a second choice set with the other. No ordering occurs in the writing or selection. In the following example, (skirt, sweater) is not different from (sweater, skirt). See figure 16.

> Amy has 3 sweaters with different patterns. She also has 5 different skirts. How many outfits consisting of a sweater and a skirt are possible?

The pairs can be determined from a matrix (table 1) or from a "factor tree." Either way, multiplication is used: 3 × 5 = 15 outfits.

Selection 2 problems result when one choice set and the number of pairs are given and the other choice set is sought. These problems are similar to selection 1 problems.

Table 1
A Matrix to Record the Pairs in Figure 16

	Skirts				
Sweaters	1	2	3	4	5
A	A, 1	A, 2	A, 3	A, 4	A, 5
B	B, 1	B, 2	B, 3	B, 4	B, 5
C	C, 1	C, 2	C, 3	C, 4	C, 5

Selection 3 problems involve triples, quadruples, or other extended n-tuples ($n > 2$) and the choice sets for each place in the n-tuple.

> Frank has 5 sport coats, 3 vests, and 5 pairs of trousers, all of which are color compatible. How many different outfits consisting of a sport coat, vest, and pair of trousers are in his wardrobe?

Here a 3-tuple must be formed (sport coat, vest, trousers) where ordering is not important. Finding the total number of 3-tuples uses the multiplication principle: 5 × 3 × 5 = 75.

Selection 4 problems give the number of n-tuples and the sizes of all but one choice set, which is sought. An example follows:

> Frank can make 24 different outfits consisting of a sport coat, vest, and trousers. He has 3 sport coats and 4 pairs of trousers. How many vests does Frank have?

Fig. 16 A *selection 1* problem: Amy has 3 sweaters with different patterns. She also has 5 skirts of different colors. How many outfits, consisting of a sweater and a skirt, are possible?

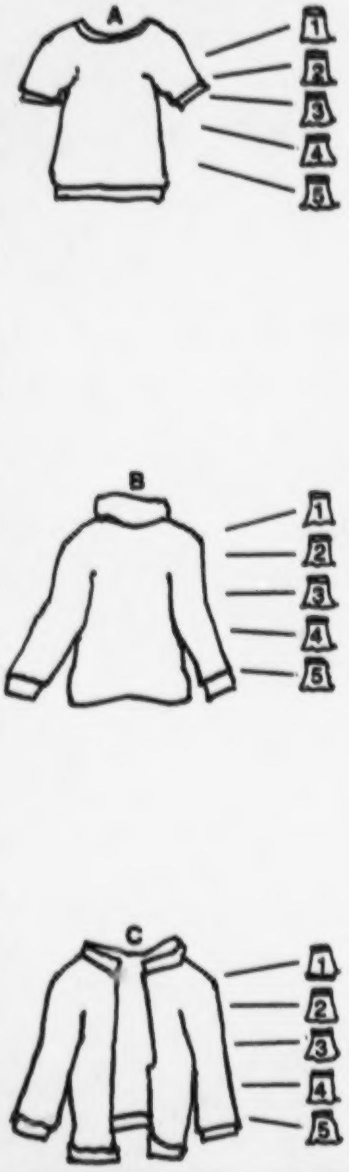


Fig. 17 Ceramic tiles can be used to link the repeated-addition idea of multiplication to area: Make 4 rows of 6 tiles each. How many tiles are used?

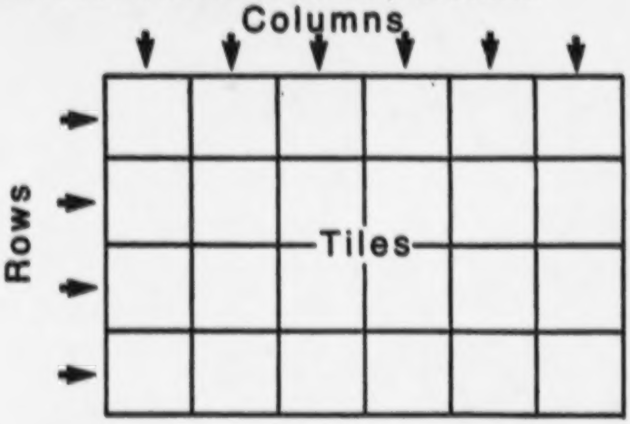


Table 2
Change Problems

Problem title	Sample problem	Characteristics
(Change 1) Repeated addition	Fred has 3 boxes with 4 cars in each box. How many cars does Fred have?	Initial (smaller) set sizes and change number known; question about final (larger) set.
(Change 2) Repeated subtraction (measurement)	Jean had 12 cookies. She gave 3 cookies to each of her friends. How many of her friends got cookies?	Initial (larger) set and final (smaller) set sizes known; question about change number.
(Change 3) Partitioning into equal sets	Paul had 24 marbles that he gave away to 4 friends. Each friend received the same number of marbles. How many marbles did each friend get?	Initial (larger) set and change numbers known; question about the size of final (smaller) sets.

This is a two-step problem: first multiply and then divide, or successively divide.

The selection group of problems involves the multiplication principle or one aspect of what Piaget calls combinatorial reasoning—the ability to consider the effect of several variables simultaneously. Selection 1 problems can be thought of as cells in a matrix. The thinking needed to solve them is similar to that used to solve area problems, such as being given two dimensions and finding the area and being given the area and one dimension and finding the other dimension.

Overview

If students are going to apply multiplication and division to everyday situations, they must have experience with materials that represent these different situations.

The change situations that involve joining and separating can be introduced with materials that can be joined, separated, and arranged. Unifix cubes can be used to illustrate repeated additions and repeated subtractions as well as measurements. Ceramic tiles can also be used to link the idea of repeated addition to area (fig. 17). The measurement concept of division can also be introduced with tiles. The following kinds of questions can be asked:

- Given 24 tiles, how many rows can be made with 4 tiles in each row?
- Make 4 rows of 6 tiles each. How many tiles are used?

Beans and paper cups can be used to give experience with the partition interpretation of division as well as to the repeated-addition and repeated-subtraction interpretations of multiplication and division. Some examples are the following:

- Given 21 beans, put 3 beans in cups until the beans are gone. How many cups did you use?
- Given 35 beans, put an equal number of beans into each of 5 cups. How many beans are in each cup?
- Given 4 cups, put 5 beans in each cup. How many beans were needed?

The *ratio comparison* situations can be introduced with two different shapes, two different colors of chips or cubes, or any other materials that can be put into sets and compared using the multiplication- and division-related questions in the examples.

The *selection* ideas can be introduced best with colored cubes or several geometric shapes in different colors, forming pairs and triples of these materials. Subsequently using situations that involve items from the students' experience, such as stickers, pizza toppings, clothing, and record labels, can help children apply these basic ideas of multiplication to the real world.

Rate problems should probably be introduced after establishing the idea of a constant rate of change in two related variables. This introduction must be done slowly and carefully and timed to the stage of cognitive development of the students. The demands are primarily on the proportional-reasoning capability of the students.

Introducing problems involving such relationships as *distance = time × rate*, *cost = cost/unit × units*, and *percentage = percent × base* should be within the more general context of rate of change. Otherwise students may substitute values into formulas without understanding the processes involved.

Table 3
Compare Problems

Problem title	Sample problem	Characteristics
Compare 1	Joellen has 3 pairs of sandals. She has 4 times as many pairs of shoes. How many pairs of shoes does she have?	Referent set and many-to-one correspondence known; question about the comparison set.
Compare 2	Irene has 30 pennies. She has 5 times as many pennies as Pat has. How many pennies does Pat have?	Comparison set and many-to-one correspondence known; question about the referent set.
Compare 3	Donald has 5 marbles. Peter has 15 marbles. Peter has how many times as many marbles as Donald?	Comparison set and referent set given; question about kind of (many-to-one) correspondence.
Compare 4	Bonnie has 16 white blouses and 4 colored blouses. Her colored blouses are what (fractional) part of her white blouses?	Comparison set and referent set given; question about kind of (one-to-many) correspondence.
Compare 5	Our class has 16 boys and 12 girls. There are how many times as many boys as girls?	Comparison set and referent set given: question about the (many-to-many) correspondence.
Compare 6	Our class has 16 boys and 12 girls. The girls are what (fractional) part of the boys?	Comparison set and referent set given; question about many-to-many correspondence.
Compare 7	Fred has 25 baseball cards. He has 5/4 as many cards as Jim has. How many baseball cards does Jim have?	Comparison set and many-to-many correspondence given; question about referent set.
Compare 8	Erica has 25 stickers. Peggy has 4/5 as many stickers as Erica. How many stickers does Peggy have?	Referent set and many-to-many correspondence given; question about comparison set.

Table 4
Selection Problems

Problem title	Sample problem	Characteristics
Selection 1	Paula has 3 kinds of cheese and 2 kinds of sausage. How many different cheese-and-sausage pizzas can she make?	Number given from which to select for each pair element; question about number of pairs possible.
Selection 2	Frank makes 18 different cheese-and-sausage pizzas. He has 6 kinds of cheese. How many kinds of sausage does he have?	Number in one choice set and number of pairs given; question about number in other choice set.
Selection 3: extended *n*-tuple	Dave has 3 different-sized sets of wheels, 4 kinds of bodies, and 3 different motors. How many different cars with wheels, a body, and a motor can he put together?	Number given from which to choose for each portion in *n*-tuple; question about number of *n*-tuples possible.
Selection 4: extended *n*-tuple	Dave has 3 different-sized sets of wheels and 4 kinds of bodies; he can make 96 different cars with wheels, bodies, and motors. How many different kinds of motors does he have?	Number given from which to choose for all but one position in *n*-tuple and also number of *n*-tuples; question about remaining position.

Table 5
Rate Problems

Problem title	Sample problem	Characteristics
Rate 1	Lisa buys 18 cans of polish at $0.72 per can. What is the total cost?	Given the rate and the independent variable value; question is about the dependent variable.
Rate 2	Peter buys a suit on sale. The price, after a 25% discount, is $90. What was the original price?	Given the rate and the dependent variable value; question is about the independent variable.
Rate 3	Corrine runs 200 meters in 72 seconds. What is her average speed in meters per second?	Given the values of the dependent and independent variables; question is about the rate.

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EXAMPLES OF DIFFERENT SITUATIONS THAT LEAD TO THE ARITHMETIC OPERATIONS

Combine 1: Tony has 10 red marbles and 12 blue marbles. How many marbles does Tony have?

Combine 2: Jack has 26 pets. 11 are dogs and the rest are cats. How many cats does Jack have?

Change 1: Joni has 9 cassettes of her favorite groups. On her birthday she received 8 more cassettes. How many does she have now?

Change 2: Joyce has 23 poppies. She sold 18. How many poppies does she have left to sell?

Change 3: Before Willie gave him some more nails, Fred had 26. Now he has 40. How many nails did Willie give him?

Change 4: Connie had 13 extra valentine stickers. She gave some to Ruth. Now Connie has only 4. How many stickers did Ruth get?

Change 5: Tom had some hazelnuts in a basket. Jerry put 19 hazelnuts into the basket. Then Tom had 34 hazelnuts. How many hazelnuts were in Tom's basket at the start?

Change 6: Gloria had some pennies in her purse. She used 8 of these to pay for some buttons. She then had 9 pennies in her purse. How many pennies were in Gloria's purse to start with?

Compare 1: Petra has 11 baseball cards. Gerta has 18 baseball cards. How many more cards does Gerta have?

Compare 2: Tomas has 9 scout badges. Willie has 19 scout badges. Tomas has how many fewer badges than Willie?

Compare 3: Walter has 8 pencils. Jeannette has 4 more pencils than Walter. How many pencils does Jeannette have?

Compare 4: Sharon has 12 campaign buttons. June has 3 fewer buttons. How many buttons does June have?

Compare 5: Bobbie got 23 correct on his spelling test. She got 6 more correct than Barbara. How many did Barbara have correct?

Compare 6: Tess did 13 push-ups in physical education class. She did 8 fewer than Vera. How many push-ups did Vera do?

CHANGE 1: Freda has 4 boxes with 5 packets of seeds in each box. How many packets of seeds does Freda have?

CHANGE 2: Johanna had 30 cookies. She gave 6 cookies to each person in her troop. How many of her friends received cookies?

CHANGE 3: Paul had 24 marbles that he put into 4 bags. He put the same number in each bag. How many marbles were in each bag?

COMPARE 1: Joellen has 4 pairs of sandals. She has 5 times as many pairs of stockings. How many pairs of stockings does she have?

COMPARE 2: Irene has 30 pennies. She has 5 times as many pennies as Pat. Pat has how many pennies?

COMPARE 3: Donald has 6 marbles. Francis has 18 marbles. Francis has how many times as many marbles as Donald?

COMPARE 4: Bonnie has 16 white blouses and 4 colored blouses. She has how many white blouses for each colored blouse?

COMPARE 5: Her colored blouses were what fractional part of her blouses?

COMPARE 6: Our class has 16 boys and 12 girls. There are how many times as many boys as girls?

COMPARE 7: Our class has 15 boys and 12 girls. There are how many girls for a group of how many boys?

COMPARE 8: The girls were what fractional part of the class?

COMPARE 9: Fred has 25 baseball cards. He has 5/4 as many cards as Bill. Bill has how many baseball cards?

COMPARE 10: Tom as 25 baseball cards. Tim has 4/5 as many baseball cards as Tom. Tim has how many baseball cards?

COMPARE 11: Jack had some marbles. Dennis had 12 marbles or 2/3 as many as Jack. Jack had how many marbles?

SELECTION 1: Paula has 3 kinds of cheese and 2 kinds of sausage. How many different cheese and sausage pizzas can she make?

SELECTION 2: Frank's makes 18 different cheese and sausage pizzas. He uses 6 kinds of cheese. How many kinds of sausage does he have?

SELECTION 3: Rita is going to make a soapbox derby car. She has 3 sets of different size wheels, 4 different boxes for bodies, and 3 different windshields. How many different cars with a set of wheels, a body and windshield can she make?

SELECTION 4: Bonnie can wear 30 different outfits consisting of a skirt, blouse and shoes. She has 3 skirts, and 5 blouses. How many pairs of shoes does she have?

RATE 1: Lisa bought 6 cans of potato chips at "2 cans for 59¢". How much did potato chips cost her?

RATE 2: Corrine ran 100 meters in 15 seconds. What was her average speed in meters per second?

LEVEL FIVE

PROBLEM SOLVING: CHECKING UP

Background: Children have had an opportunity to see all of the addition and subtraction kinds of problems and the change and combine multiplication and division problems. Give the problem solving "checking up" test. Analyze the results and determine which kinds students need more experience with.

LESSON ONE: Making Sure

Review with children the use of "less than," "more than," "fewer than," "times as many", "part of", as these ideas occur in different problems. Encourage children to think of questions about problems:

"What does the question ask for?"
"What does the bigger number(s) show?"
"What does the smaller number(s) show?"
"Is something being compared with something else?"
"Is something being added or subtracted?"
"What is not known?"

Encourage children to make diagrams to represent quantities.

Problem	Number Sentence	Converted Sentence
Combine 1:	A + B = □	
Combine 2:	A + □ = C	□ = C - A
Change 1:	A + B = □	
Change 2:	C - A = □	
Change 3:	A + □ = C	□ = C - A
Change 4:	C - □ = B	C + B = □
Change 5:	□ + B = C	□ = C - B
Change 6:	□ - B = A	□ = A + B

Many children will mentally convert to the model they are most comfortable with. If you encourage them to make pictures to represent the objects in the problems, they are more likely to write correct number sentences - either direct models or conversions. Most of the COMPARE problems will yield proper number sentences this way.

LESSON TWO: Emphasis on Modelling

Introduction: In this lesson go through the problem types one at a time. Use materials - base ten blocks, counters or whatever is most appropriate. Make a diagram of each type. Write the number sentence, explaining how each numeral and symbol is derived from the verbal description.

A CHANGE 3 problem is used as an example.

"Betty has 12 star coupons. Here are her coupons."

☆☆☆
☆☆☆
☆☆☆
☆☆☆

Write

12

"Fred gives Betty some more coupons." Write: $12 + \boxed{}$

"Why did we write the plus sign?"

"Why did we write the $\boxed{}$ sign?"

Turn overhead off. Add six more counters to the collection. Turn overhead on.

"Betty now has 18 of these."

☆☆☆
☆☆☆
☆☆☆
☆☆☆
☆☆☆
☆☆☆

Write

$12 + \boxed{} = 18$

"How many did Fred give Betty?" Write: $12 + \boxed{6} = 18$

Circle the answer: $12 + \enclose{circle}{\boxed{6}} = 18$

"SIX is the missing amount that we didn't know - the number Fred gave Betty."

LESSON THREE: Writing Problems

Introduction: Write a number sentence on the overhead or chalkboard:

$$23 + \square = 44$$

"Write a story problem so that this number sentence shows what is in the problem."

Ask for these from individuals and analyze them as a group. Remind children "+" can show parts or the action of joining. It may also show "how much more than." The $\square$ always shows the number to be found - the answer to the question in the problem.

Activity: Pass out the worksheet and have children write a story problem for each. Allow the use of materials as needed.

LESSON THREE: Checking Up 2

Background: Children have had enough experience with some of the problems requiring multiplication and division to warrant finding out which of these need instructional emphasis. Read the material in the APPENDIX first.

Introduction: Each child should have 30 cubes of some kind and a calculator. Remind them to use these to represent objects in the problems. "Listen as I read the problem and show it to you. First, find the answer. I will read it a second time. This time concentrate on the number sentence to show the problem. Then CIRCLE the number in the sentence that answers the question in the problem."

Reveal the problems one at a time. Read each one twice slowly. Give children time to THINK between each step. Analyze the results to determine which kinds of problems to emphasize.

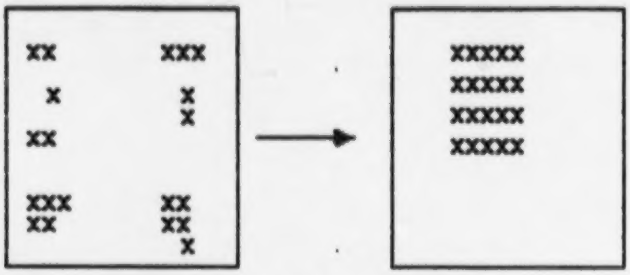
LESSON FOUR: Multiplication and Division

Introduction: Go through the problem kinds for multiplication and division one at a time. Follow the same procedure as with addition and subtraction.

Allow the use of materials
Emphasize writing number sentences

Emphasize making diagrams to show the problem situations

CHANGE 1: "I put 5 pictures on each of 4 bulletin boards. How many picture did I use?" Materials to show this on the overhead:

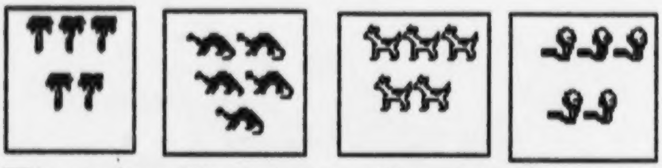


"There are 20 pictures altogether."

Number Sentence:

$\boxed{} = 4 \times 5$ **"20 goes in the box."**

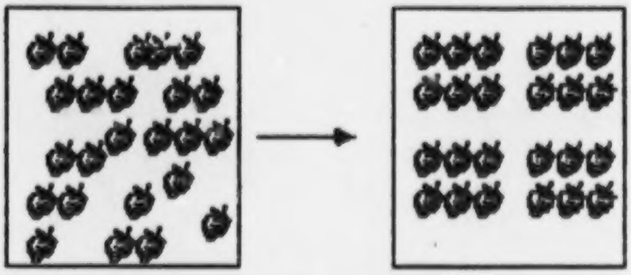
Picture:



"There are 20 pictures altogether."

CHANGE 2: Joyce had 24 strawberries.. She gave 6 to each of her friends. How many friends received strawberries? Materials to show this on the overhead:

Sort out by 6's:



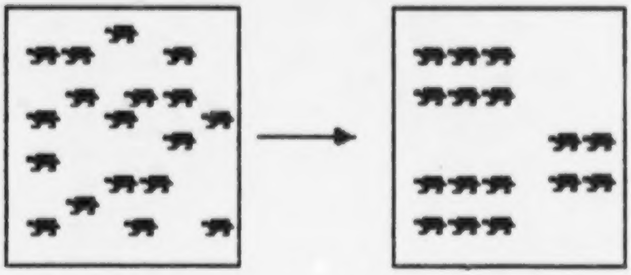
"There are 4 collections of 6, so 4 friends received strawberries."

Number Sentence: $24 \div 6 = \boxed{}$

"4 goes in the box."

CHANGE 3: George had 18 turtles. He gave an equal number to each of 3 friends. How many turtles did each friend get? Materials to show this on the overhead.

Sort into 3 groups one at a time:

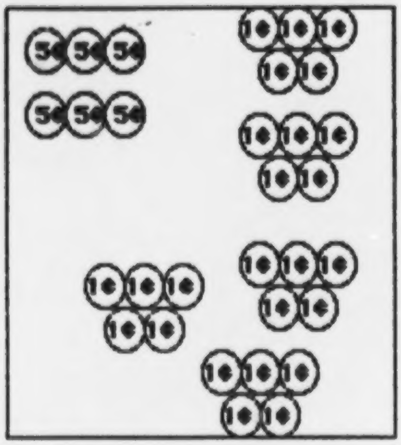


"Each of the 3 friends received 6 turtles."

Number Sentence:

$\boxed{} = 18 \div 3$ "6 goes in the box."

COMPARE 1: Francine has 6 nickels. She has 5 times as many pennies as nickels.. How many pennies does Francine have? Materials to show this on the overhead:

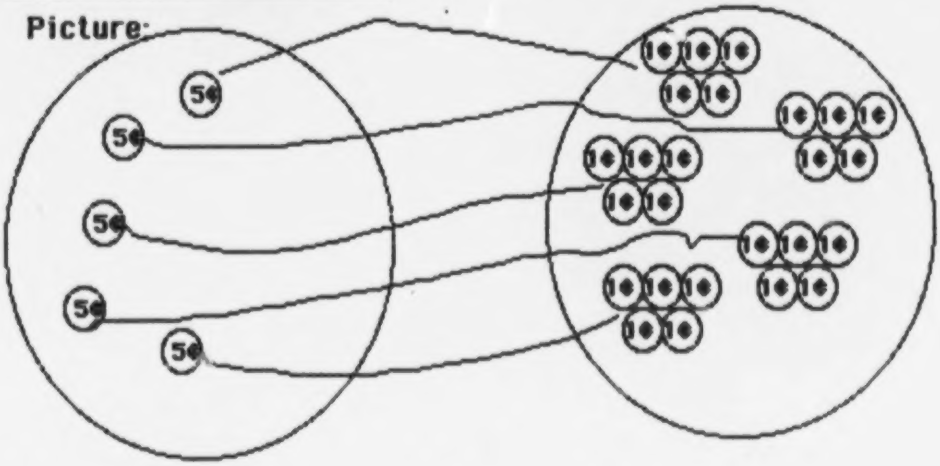


"For each nickel, there are 5 pennies. For 2 nickels there are 10 pennies. For 6 nickels, there are 30 pennies."

Number Sentence: 5 x 6 = ☐

"30 goes in the box."

Picture:

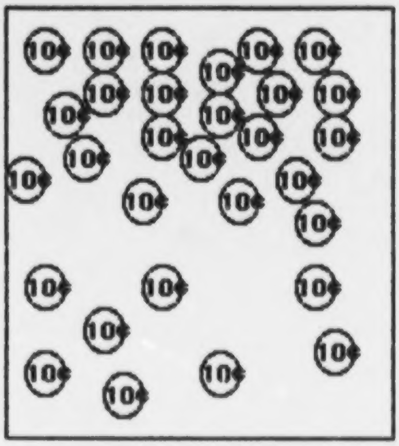


"For each nickel there are 5 pennies."

TABLE:

Nickels	Pennies
1	5
2	10
3	15
4	20
5	25
6	30

COMPARE 2: Jane had 30 dimes. She has 3 times as many dimes as Pat. How many dimes does Pat have? Materials to show on the overhead:


10¢
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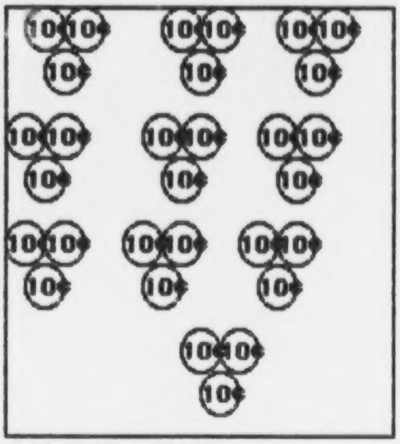
10¢

"Jane has 3 dimes for every one of Pat's."


10¢
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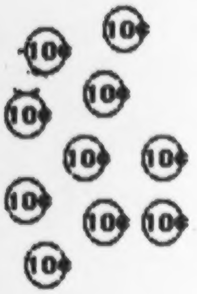
10¢

"There are 10 groups of 3 so Pat has 10 dimes."

PICTURE: PAT JANE



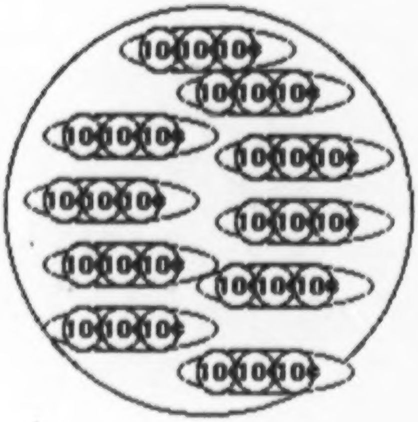
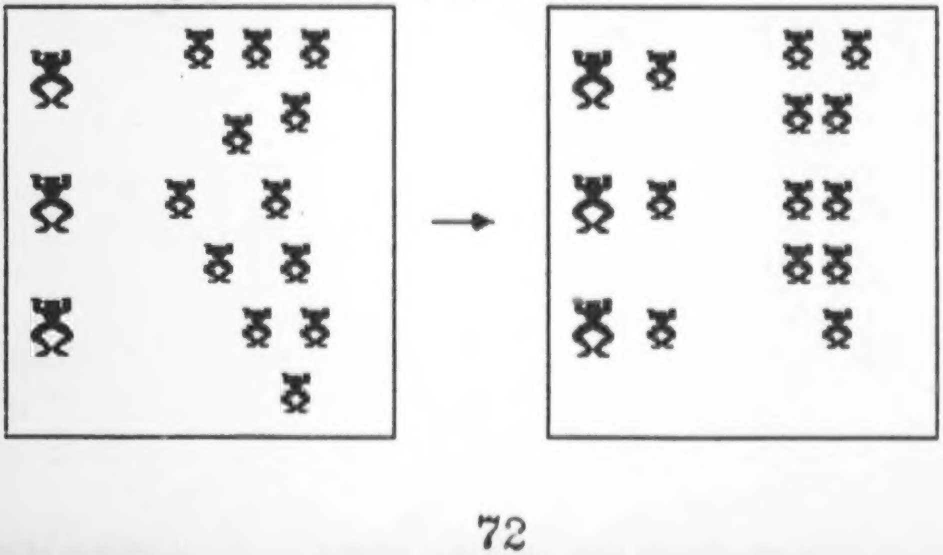


TABLE:

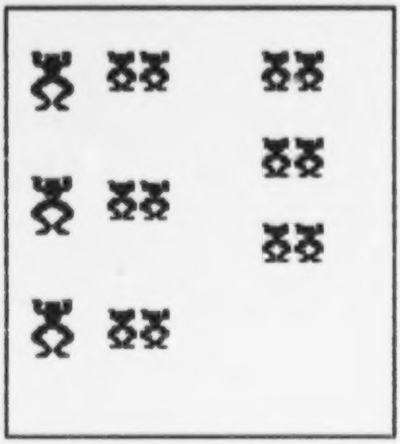
Pat	Jane
1	3
2	6
3	9
4	12
5	15
6	18
7	21
8	24
9	27
10	30

COMPARE 3: Davey has 3 frogs. Paul has 12. Paul has how many times as many frogs as Davey? Materials on the overhead.

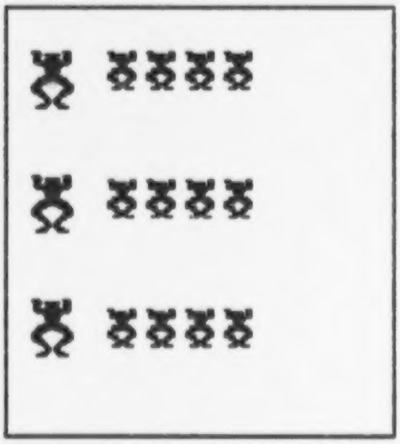
Let's arrange these first as one of each:



"We can use another so there are 2 for each."



"We can give each large frog 2 more, so:

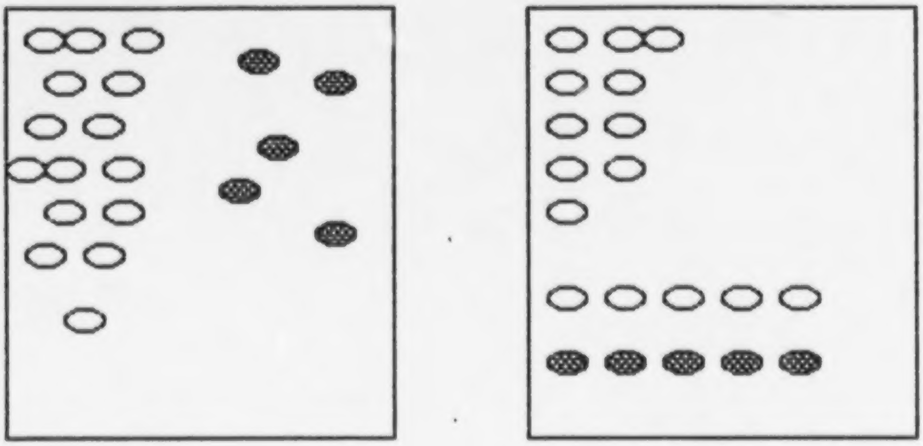


There are 4 for each one."

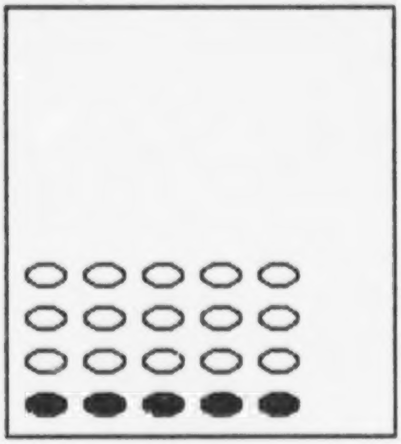
Number Sentence: $\boxed{} = 12 \div 3$ "4 goes in the box."

COMPARE 4: John has 15 pairs of white sox and 5 pairs of colored sox. He has how many pairs of white sox for each pair of colored sox? Materials to show on the overhead projector:

Let's arrange these first using 1 for 1:



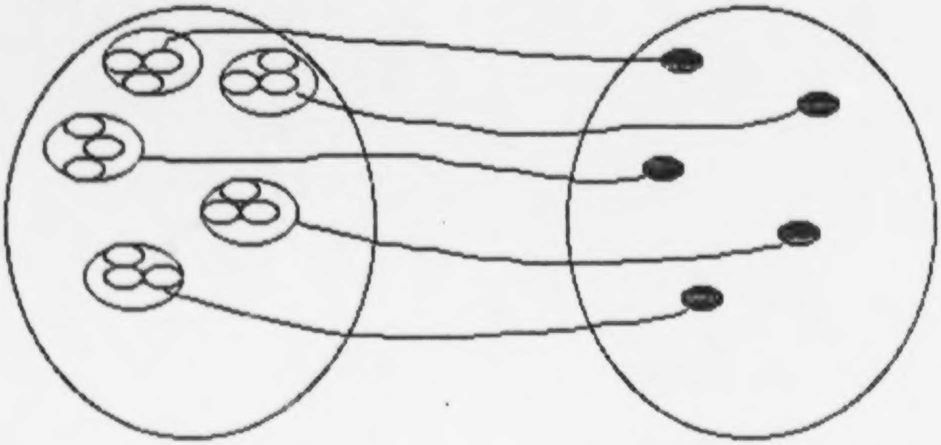
"How many more whites can we put with each colored one?"



"There are 3 whites
for each colored one.."

Number Sentence: 15 ÷ 5 = ☐ "3 goes in the box."

Picture:



COMPARE 5: His colored sox are what fractional part of all of his sox?

"How many sox does he have?" 20
"How many colored sox does he have?" 5
"5 is what part of 20?" 1/4

It is unlikely your assessment will yield many students who can handle many-to-many comparisons as in COMPARE 6-11. The ratio work that you do with that set of lessons will give the background needed so those kinds of problems can be worked on at the next level.

LESSON FIVE: More Multiplication & Division

Introduction: This is to introduce children to the selections problems that lead to multiplication and division.

Selection 1: Materials to use on the overhead.

"Gloria has 3 dogs and 4 collars for these. In how many different ways can she match a dog with a collar?"

"Let's represent these. We'll let colored chips represent the collars and different shapes represent the dogs:

△ is Fido

□ is Spot

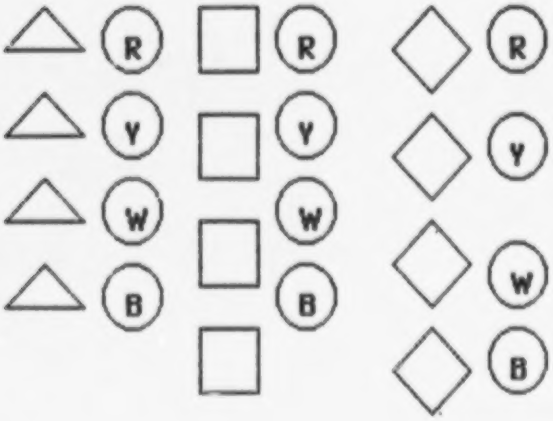
◇ is Rover

Ⓡ Ⓨ Ⓦ Ⓑ are the collars."

"How can we match the collars of different colors with the dogs.:

△ Ⓡ
△ Ⓨ "Fido can wear a collar 4 different ways."
△ Ⓦ
△ Ⓑ

△ Ⓡ   □ Ⓡ "Spot can wear one of these collars 4 different ways."
△ Ⓨ   □ Ⓨ
△ Ⓦ   □ Ⓦ
△ Ⓑ   □ Ⓑ
□

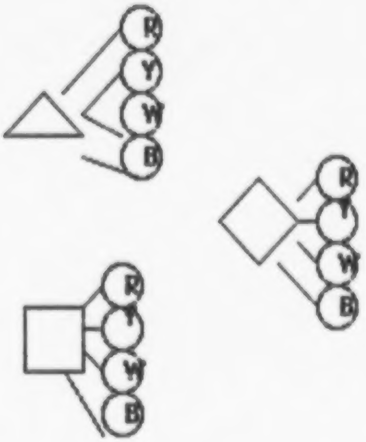

R
Y
W
B
R
Y
W
B
R
Y
W
B

"Rover can wear one of the collars in 4 different ways, also."

"How many matchings do we have?"

"What are some ways that we could show this in a table or a diagram?"

		Dogs		
		△	□	◇
Collars	Ⓡ	1	2	3
	Ⓨ	4	5	6
	Ⓦ	7	8	9
	Ⓑ	10	11	12


R
Y
W
B
R
Y
W
B
R
Y
W
B

Number Sentence: □ = 3 x 4

Selection 2: In these the total number of pairs is known and the number of things possible for one part of the pair is known, so division is neded.

"Frank can make 15 outfits including a sweater and a pair of slacks. He has 3 sweaters to choose from. How many slacks does he have?"

Let's look at a collection of pairs of UNIFIX cubes and try to put them into an array by classifying them."

Have fifteen UNIFIX pairs:
 5 with red followed by white, green, blue, brown or black;
 5 with yellow followed by white, green, blue, brown or black;
 5 with orange followed by white, green, blue, brown or black

These should be in random arrangement.

"How should we classify these in an arrangement of rows and columns?"

You should eventually get the following:

R	W	R	G	R	Bl.	R	Br.	R	Bk.
Y	W	Y	G	Y	Bl.	Y	Br.	Y	Bk.
O	W	O	G	O	Bl.	O	Br.	O	Bk.

"The first UNIFIX in each pair is the same for every row and the second is the same for every column."

If we knew the 3 colors used first or the 4 colors used second, we could find the other one from the number of pairs. In this case: 12 ÷ 3 = 4 or 3 = 12 ÷ 4.

"You tell me what to do to work this problem."

"Tim has 15 hat and scarf combinations. He has 3 hats. How many scarves does he have?"

Activity: Give students some of the provided problems to work on. Allow the use of materials to represent objects in the problems. Have them write number sentences and circle the number that answers the question.

LESSON SIX: Two-step Problems

Background: These problems involve the application of one arithmetic operation followed by another. To do these, children MUST very thoroughly understand the situations that give rise to the four arithmetic operations.

Introduction: "Tom has 8 bags with 8 marbles in each bag. He has another bag with 26 marbles in it. How many marbles does he have in all?"

"Let's look at this one step at a time." "How many marbles are in all of the 8 bags?" 64.

"He has another bag with 26 marbles. What do we do with this 26?"

"How many marbles do we have in all?"

"Notice we multiplied first, then added something to that result."

Write: $\boxed{} = 8 \times 8 + 26$

Do a second:

"Sally put 9 coins on each of 7 pages in her coin collection book. She had 77 other coins in a box. How many coins did she have in all?"

"What do we do first?"

"How many coins is that?"

"How do we get the number of coins IN ALL?"

"What is the number sentence?'

$\boxed{} = 9 \times 7 + 77$

Activity: Pass out the activity sheets and monitor the work closely.

LESSON SEVEN: Non-Traditional Problems

Background: Problem solving does not all fall neatly into cases of adding, subtracting, multiplying and dividing, or combinations of these. Some problems require use of other strategies. These include:

Guessing and Checking
Making a picture
Making a table
Making a list
Finding a pattern(s)
But mostly, THINKING

Introduction: Work two or three problems that illustrate these. Some examples are:

"Sam has 7 coins, all nickels and dimes. Their total value is 50¢. What are the 7 coins?"

"Could all 7 be dimes? Why not?"

"Could all 7 be nickels? Why not?"

"What amounts could be in dimes?"

List these as given:

Dimes:
10¢
20¢
30¢
40$

"What amounts could be in nickels?"

Discuss why they must all end in "0".

List as given:

10¢
20¢
30¢
40¢

Make a table:

No. of Dimes	Amount	No. of Nickels	Amount
①	10¢	2	10¢
2	20¢	$\boxed{4}$	20¢
$\boxed{3}$	30¢	⑥	30¢
4	40¢	8	40¢

"How can we get 7 coins?" Circle with colored chalk.

"Which gives 50¢?"

So, we have 3 dimes and 4 nickels.

"The time is 7:35. What time will it be in one hour and twenty-five minuts?"

"What is the time are one hour?" 8:25

"What is 25 minutes added to this time?"

"So, it will be 8:50."

"Janis is thinking of two numbers. If she adds the two numbers, it is 17. The larger is 9 more than the smaller. What are the numbers?"

"What are some ways to do this?" Possibilities include looking at all 2 number combinations making 17. Another is looking at numbers 9 apart.

Possibility 1:
1. 16
2. 15
3. 14
4. 13
5. 12
6. 11
7. 10
8. 9

"How many of these pairs have larger 9 more than smaller?

Possibility 2:
1. 10
2. 11
3. 12
4. 13
5. 14
6. 15
7. 16

"How many of these add to 17?"

The result is clearly 4 and 13.

"How much do 30 eggs cost if eggs are 66¢ per dozen?"

"What must we do first?" Pick up on suggestion to see how many dozen in 30:

$$30 \div 12 = \square$$
$$\square = 2\ 1/2$$

"Now what must we do?"

"Let's summarize all we did in a number sentence."

$$30 \div 12 \times 66¢ = \square$$

"Could we have done this another way?" Make a table:

	Eggs	Cost	
1	6	33	
	12	66	What we know
2	18	99	
3	24	132	
4	30	156	What we need

Activity: The attached set of problems should be used as a source for giving problem solving work at least once a week.

Sometimes have the children work on these in pairs. Sometimes work with the whole group in analyzing and thinking through a problem. Sometimes have children list all of the different ways to work a given problem after having worked it.

LESSON EIGHT: Creating Problems

Introduction: Explain to the children that they will be given some information, but no question about it. They are to write as many questions about this that require a number as they can. Give one example:

> "The flag of the United States has 7 red stripes and 6 white stripes."

Try to get as many related questions as you can - for example:

> "There are how many more red stripes?"
> "How many stripes are on the flag?"
> "The number of white stripes is what fraction of all of the stripes?"

Activity: Use the problem sheets. Have children write questions about the information given.

LEVEL FOUR

ASSORTED PROBLEMS TO USE

1. As a problem of the day
2. For assignments
3. For pairs of children to work on, etc.

1. Bridgeman's is having an ice cream cone special. They have cake cones and sugar cones. They will let you choose from chocolate, vanilla or strawberry ice cream. You can add crushed nuts or marshmallow topping. How many different choices of an ice cream cone do you have?

2. In preparing for a marathon, Frank ran every day of the week for a total of 100 miles. He ran 13 miles on Saturday and 12 miles on Sunday. On the rest of the days he ran the same number of miles each day. How many miles did he run on the weekdays?

3. Several children in the class have dogs as pets. There are three times as many Collies as Spaniels. There are 6 more Spaniels than poodles. Three children have poodles. How many children have dogs?

4. June sold 28 paperbacks for 10¢ each at her mother's garage sale. She has 27 paperbacks left. She wants to get a total of $10.00. How much should she charge for each remaining paperback?

5. Dean used pattern block triangles to make large triangles. The largest triangle required 36 triangles. The side of this triangle is how many times as long as the side of a pattern block triangle.

6. Vicki, Shelley, Tom and Chris each have a dog. The dogs are Spaniel, Spitz, Terrier and Labrador. Vicki has the Spaniel; Shelley does not have the Spitz or the Labrador. Chris does not have the Labrador. Who has which dog?

7. Al's 3 stage rocket model is 120 centimeters long. With the first stage removed, it is 74 cm. With the second stage

removed as well, it is 42 cm. long. Which stage is the longest? How much longer is it than each of the other two?

8. Linda paid for her records with $2.25 in quarters and dimes. She used 2 more quarters than dimes. What coins did she give the clerk?

9. Terri's mother made a cake in a pan 9 in. x 12 in. When it cooled, she frosted it and decided to put chocolate soldiers around the top edge of the cake. If she put the soldiers 3 inches apart, how many did she use?

10. Of the 30 players on the football team, all but 2 are going to the awards banquet. They will be seated 4 to a table. How many tables are needed?

11. Tabitha numbered the pages of her diary. It has 150 pages. How many times did she use the digit "4"?

12. Lynn, Iris and Sue found a boxful of marbles. They set aside half of them to use later. Each girl took 1/3 of the rest of the marbles? Lynn received 12 marbles. How many marbles were in the box?

13. Larry went smelting. Each time he dipped the net, he had 2 more smelt than the last time. How many smelt were in the net when he dipped it the tenth time?

14. The 4th grade class has 5 gerbils and 2 cages. In how many different ways could they be put in the cages, without any cage being empty?

15. Phyllis's mother went on a diet for 30 days. To make it more challenging, Phyllis's father said he would pay her $2.00 for every day she lost weight, but would charge her $1.50 for every day she gained weight. At the end of 30 days, Phyllis's father paid her $25. On how many days did Phyllis's mother lose weight?

16. Denny made a deal with his neighbor to mow his lawn. He said he would charge $4.00 for each of the first 5 times and $5.00 for each time more than 5. He mowed the lawn 12 times. How much did his neighbor pay Denny?

17. Shelley and Gregg went to the movie. It started at 6:30. Previews of coming attractions took 12 minutes. Commercial ads took 7 minutes. The film lasted 1 hour and 30 minutes. Their bus left at 8:30. How long did they have to catch the bus?

18. Tickets for the football game were numbered 500-1000. Each person having a ticket with only one 6 and no other digit smaller than 8 received a free banner. How many banners were given away?

19. Fritz tried to make the longest UNIFIX link in the class. He used 900 UNIFIX cubes. Each is 3/4 inch long. How long was his UNIFIX cube?

20. Red pencils are 3 for 89¢ and yellow pencils are 4 for 89¢. If Tom's bill was $5.34 and he bought more yellow pencils than red pencils, how many pencils did he buy?

21. 5 pound bags of potatoes cost 89¢ and 10 pound bags cost 1.59¢. How much cheaper is it to buy all 10 pound bags if you need 50 pounds of potatoes?

22. 132 people attended Rocky VIII. Adult tickets cost $4.00 and children's tickets are $2.50. How many children attended if the total receipts for tickets were $480?

23. Two-thirds of Mrs. Runions class are boys. To even things, 5 boys go to Mrs. Davern's class and 5 girls come to Mrs. Runions' class. Now only one-half of Mrs. Runions' class are boys. How many students are in Mrs. Runions' class?

24. Write 5 division problems that have an answer 7 R 3.

25. Gerte is now 7 inches taller than her brother Dean. She grew 3 inches last year while Dean grew 4 inches. A year ago Dean was 4 feet 3 inches. How tall is Gerte?

PROBLEMS TO USE ON ASSIGNMENTS FOR STUDENTS

These are also models of kinds of multiplication and division problems for you to use in writing more of these.

"Bridgeman's has 10 flavors of ice cream and 5 different toppings. How many different ice cream cones can be made?"

"I put 5 pictures on each of 4 bulletin boards. How many pictures did I use?"

"I have 6 bags of apples with 9 apples in each bag. How many apples do I have?"

If 7 boxes of crayons have 8 crayons in each box, how many crayons are there?"

"Bill has 9 pennies. His brother has 4 times as many pennies as Bill. How many pennies does his brother have?"

"Fred's marble bag has only 4 marbles in it. John's bag has 7 times as many marbles. How many marbles are in John's bag?"

"Jane has 18 crackers. She put them into piles of 6. How many piles did she have?"

"If 24 apples are put into 3 bags so that each bag has just as many apples, how many will be in each bag?"

"John caught 3 times as many perch as sunfish. He caught 15 perch. How many sunfish did he catch?"

"Bill's box has 18 washers and 6 nails. He has how many times as many washers as nails?"

"Paula has 24 different sweater and skirt outfits. She has 6 sweaters. How many skirts does she have?"

"How many 6-packs of pop do you have to have to buy so each one in a class of 54 can have one bottle?

"If 5 tires are sold with each car, how many tires in all have to be supplied with 8 cars?

"Jet planes have 4 engines on each plane. How many engines are there on 8 of these planes?"

"A wren weighs 4 ounces. A crow weighs 9 times as much. How much does a crow weigh?"

"John has 6 shirts and 4 pairs of trousers. How many different outfits of a shirt and a pair of trousers can he wear?"

"Jean has 4 packages of jacks. There are 16 jacks in each package. How many jacks does she have?"

"If the hockey team plays 2 games a week for 3 months, how many games will it play?"

"A chess set has 4 castles. How many castles are in 9 sets?"

"John bought 5 bags of marbles. Each bag had 12 marbles. How many did he buy?"

"Jean has 6 baseball cards. Tom has 8 times as many. How many baseball cards does Tom have?"

"72 eggs are put into cartons with 12 spaces. How many cartons are used?"

"64 pencils are put into boxes with an equal number of pencils in each box. 8 boxes are used. How many pencils are in each box?"

"If 72 straws are put into bundles of 8 straws, how many bundles are used?"

"63 marbles are put into 7 bags so each bag has just as many marbles. How many marbles are in each bag?"

"If you have 54 peanuts and give 5 peanuts to each squirrel, how many squirrels will be fed?"

"A classroom has 56 hamsters in 8 cages. How many are in each cage if they are equally divided?"

"32 Boy Scouts were divided into 4 patrols. Each patrol had just as many Scouts. How many boys were in each patrol?"

"Jean works at the deli after school. She put 48 cans of peaches in boxes with spaces for 8 cans. How many boxes did she fill?"

"If 72 apples are put into 8 bags with the same number in each bag, how many apples are in each bag?"

"How many 20 cent candy bars can you buy for one dollar?"

"How many weeks are there in 49 days?"

"What is the cost of six 8 cent pencils?"

"How many hours are there in six days?"

"How many apples costing 20¢ can you buy for 80¢?"

"In cleaning up the playground, 34 students worked on Thursday and 48 students worked on Friday. How many students worked on the two days?"

"The students picked up 432 cans and 172 bottles. How many more cans than bottles were picked up?"

"In starting a school garden, the sixth grade bought 7 packets of seed, 15 tomatoe plants and 12 pepper plants. How many things did they buy?"

"Janet picked 14 tomatoes after school. Her sister picked 9 more than Janet did. How many tomatoes did her sister pick?

"Terri had some beads for a necklace. When Elly gave her 15 more, she had enough to make a 34-bead necklace. How many beads did Terri have to start with?"

"Peter has 12 cassette tapes that play for 60 minutes each. How many minutes would it take to listen to all of the tapes?"

"A factory has 260 chairs. One store ordered 144 chairs. Another ordered 152. How many more chairs does the factory need to fill both orders?"

"A carton holds 15 screwdriver sets. Each set has 6 screwdrivers. How many screwdrivers will be sent out in 8 cartons?"

"If the school store has 120 pencils at the beginning of the week and 63 at the end of the week, how many pencils were sold during the week?"

"If the school store sold 29 green binders and 34 blue binders, how many binders were sold in all?"

"If the store sold 93 notebooks in April and 134 notebooks in May, how many fewer were sold in April?"

"The football team agreed to share the cost of the hamburgers and cakes equally. If these totalled $17.92 and 11 players shared the cost, how much did each pay?"

"If 45 seats are in 5 equal rows, how many seats are in each row?"

"How many cookies would each person get if a bag of 144 cookies was shared by 24 students?":

"How many oranges are in 6 crates that hold 86 each?"

SOME PROBLEMS TO USE FOR GROUP SOLVING AND DISCUSSION

Problems without Questions: Children supply Questions

1. Frank bought 5 pairs of skates for a total of $200.00. He sold them for $65.00 a pair.

2. Mrs. Peterson can make 10 quarts of strawberry/rhubarb sauce at a time in her soup kettle. This past summer she canned 6 gallons of sauce.

3. A recipe for cookie dough is enough to make 3 dozen cookies. Joan's sister made a 4-recipe batch.

4. The following price list is at the sporting goods store:
   - Hockey sticks
   - Hockey gloves
   - Hockey helmet
   - Hockey shin pads
   - Hockey shoulder pads
   - Hockey skates

5. Tina and Vera play basketball. Here is their scoring so far:

	Game 1	Game 2	Game 3
Tina	15	17	14
Vera	21	18	19

6. Jean sold 9 boxes of Christmas greeting cards last week. She paid $1.50 for each box and sold them for $12.25

7. Peter has 4 pairs of slacks and 3 sweaters. He can wear any sweater with any pair of slacks.

8. Ronald runs for 10 miles each Monday, Wednesday and Friday, and 15 miles each Tuesday, Thursday and Saturday.

9. A class ordered soft drinks for a picnic. They ordered twice as many cokes as root beer and one more orange drink than Pepsi. There are 12 cola drinks and 7 root beers.

10. When Jim applied for a job, there were several others who also did. The Hardee's manager said, "Show me how to make change for a 25¢ piece without using pennies."

PROBLEMS TO SOLVE AS A GROUP EFFORT

1. When John bought a new house, he found a place with a perimeter of 18 ft. where 20 tiles were missing. If each tile is a square foot, how many tiles were missing?

2. When Jacqueline went to work as a babysitter, she agreed to pay of $5 plus a bonus of $1/hour for each hour spent. The Jacksons went out for 6 hours one night. How much was Jacqueline paid?

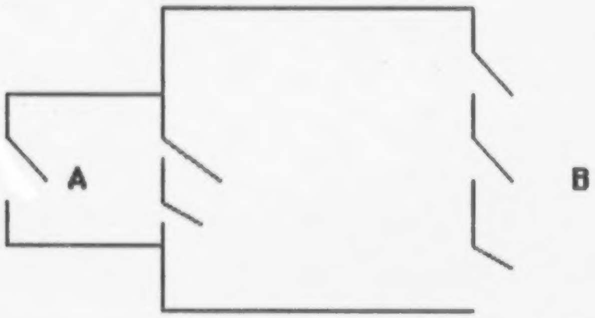
3. Time found that 4 out of every 10 kites made by a certain company have one cracked strut. How many cracked struts would he find in a shipment of 250 kites?

4. Theresa has 10 pages of science to read before class. If each page has an average of 40 lines and each line an average of 10 words, will she have time to read these 10 pages if she reads at an average rate of 30 words per minute?

5. Tom is learning to play a new guitar. He had learned 3 songs on his friend's guitar. At the end of the first week with his own guitar, he could play 5 songs; at the end of the second week, he could play 8 songs; at the end of the 3rd week he could play 12 songs. If he continues to learn new songs at this rate, how many will be know at the end of the 10th week¿

6. Fritz has $15 to buy pop for a party. If Pepsi costs $3/case and Coke $4/case; if he buys at least one case of each, how many cases of each does he buy?

7.



How many different ways are there to get from A to B using the doors?

8. Tom bought 8 erasers marked "2 for 25¢". How much did he spend?

9. Joanne saves 50¢ of every $3.00 she earns. If she earns $24.00, how much does she save?

10. On Loon Lake, there are 5 motorboats for every 3 canoes. On a Sunday, Tom counted 25 motorboats. How many canoes were on the lake?

PROBLEMS TO DETERMINE OPERATIONS TO USE

1. Paul climbed up the tree to get as close to the top as he could. How much more must he climb?

2. Susie ate some cookies and shared what was left equally with her friends. How many cookies did each friend get?

3. Tom's dog weighs 3 1/2 times as much as Louise's cat. How much does Tom's dog weight¿

4. John bought several cases of soft drinks having the same number of cans in a case. He and his friends drank some. How many cans were left?

5. Lewis runs a mile in several minutes. Franko takes longer to run a mile. How much longer does it take Franko?

6. The Bears won some games; lost some games and tied some games. How many games did the Bears play?

7. Bananas are on sale for so much per lb. How much did Tom's mother pay for a given number of pounds?

8. The earth is a given distance from the sun. Mars is a greater distance from the sun. How much closer to the sun is the earth?

9. Empty bottles are sorted out into bottle cases. How many cases are needed?

10. Ike's bicycle got a flat halfway between Pottsville and Center City. How far had Ike travelled?

PROBLEMS TO USE - MISSING INFORMATION

1. John sold both wild rice and blueberries. He sold wild rice for $4.00 per pound and blueberries for $2.00 per quart. How much did John make?

2. Paul can cut a lawn in 3 hours. How many lawns did Paul cut?

3. Grace made ten dozen cookies to sell. How much did Grace receive for her cookies?

4. Tom spent $6.89 at the store for school supplies. How much change did he receive?

5. Jean played 5 casettes during one week. How much time did she spend listening to cassettes?

6. All horses have four legs. How many legs are on all of the horses in the corral?

7. A school bus can carry 48 passengers. How many buses will be needed for the First Elementary School?

8. John had 50 Indianhead pennies in a box. He added some more. How many did he add?

9. Frances took her change as a dime, a nickel and 3 cents. How much did the pencil cost?

10. Thirty students took out library books. How many did not return their books?

PROBLEMS TO USE - USING APPROPRIATE DATA

1. If six Huskies pulling a sled can travel 24 miles per hour for 12 hours each day with a load of 600 pounds, how many miles to they average in a day?

2. Tom raised bunnies. He sold 6 of those he raised for $24 apiece. He had 8 left to sell. How many bunnies did he raise?

3. Joyce bought 6 cartons of milk. She paid $3.00 for these. Two of the cartons were sour. How much did each carton cost?

4. Eileen read the sign as: "Coffee $2.69
   Eggs $ .55
   Butter $1.29
   Milk $ 2.09"

   She bought milk and eggs. How much did she spend?

5. Fred bought a shirt for $4.00, a sweater for $15.00, and 6 pairs of stockings at $1.98 each. He returned 3 pairs of sox. How much money did he get back?

6. The Boys Club sold magazine subscriptions to raise money. They sold 90 subscriptions and made $1.50 on each subscription. They also sold books costing $5 or $9 each. How much money did they raise through selling magazines?

7. Popcorn is sold in 10 pound drums for $30.00. After being popped it is sold in bags costing 35¢. How much do 8 bags of popcorn cost?

8. Here are the savings for the Jones brothers:

Tom -	$6.95
Bob -	$6.25
Sam -	$7.15
Harry -	$5.90

   Bob saved how much less than Tom?

9. Joanne had 24 popsicles. She ate 3, 4 melted, and she gave 12 away. How many popsicles did Joanne start with?

GRAPHS

Interpreting tables, graphs and charts is important in the use of mathematics. These lessons give experience in constructing, interpreting and using these.

LESSON ONE: Graphing Data

"Last year the class kept a record of the days when snow fell. They found the following."

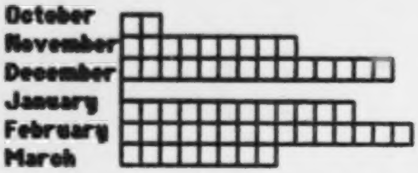
Write on the board or overhead:

October	2
November	9
December	14
January	12
February	15
March	8

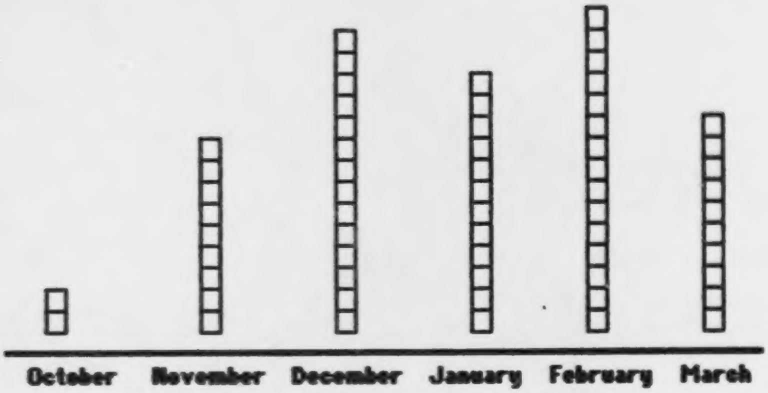
"How can we show this information in a graph?"

Discuss graphs; their components, etc. The responses to the question and discussion ensuing should result in several graphs. Show all of these.

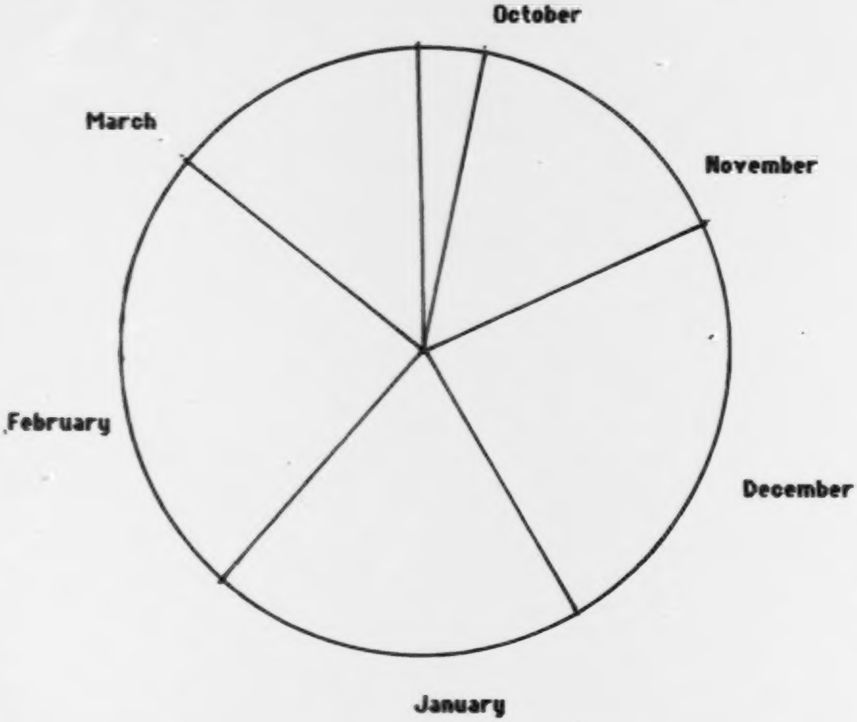
BAR GRAPH (horizontal)



VERTICAL BAR GRAPH



CIRCLE GRAPH



Use approximation to get these. There are 60 days of snowfall. 1/4 are in February; 1/5 in January, etc.

Pass out the worksheets. Have children make bar graph and circle graph for each set of data.

LESSON TWO: Interpreting Graphs

Make an overhead transparency of the graph provided. Place this on the overhead.

"This is a graph of the favorite soft drinks of a group of students. Let's answer some question s about this graph and the information on it."

"Which is the most popular drink?"
"Are any drinks of equal popularity?"
"Which is the least popular drink?
"Is there a third most popular drink?"
"Do the children preferring orange or white soda equal those preferring cola?"
"Twice as many children prefer cola as prefer what drinks?"
"More than twice as many children prefer cola as what other drinks?"

Use the transparency of the circle graph provided:

"Which is the most popular cereal?
"Which is the least popular cereal?"
"Do more people prefer hot cereal or cold cereal?"
"What is the popularity of the cereals in order? from most popular to least popular?"
"How could we put this same information into a bar graph?"

The latter question will take some time. The parts will have to be totalled in order to get some comparison between the parts. Angle measurement may have to be introduced so numbers can be assigned to the parts.

LESSON THREE: Using Tables

Put an overhead transparency of Table 1 provided on the projector.

"This is a table of the paper loops a student added to make a Christmas tree chain. He worked for seven days and stopped."

"How many loops were in the chain at the end of the 4th day?"
"How many loops more did he add each day than he had added the day before?"

"If he continued the chain for 2 more days, how may loops would be in the chain?"
"Can you write a formula for finding the length of the chain when you know how many days he worked?"

Now put a transparency of Table 2 on the overhead.

"How do we find what to enter for 'miles travelled'?"
"How do we determine miles per gallon?"
"What was total miles driven in this period?"
"What was the total number of gallons of gas consumed"
"What was the AVERAGE number of miles per gallon?"

Ask the class to find graphs and tables in newspapers and magazines to bring to class. Use these for group activity for interpretation.

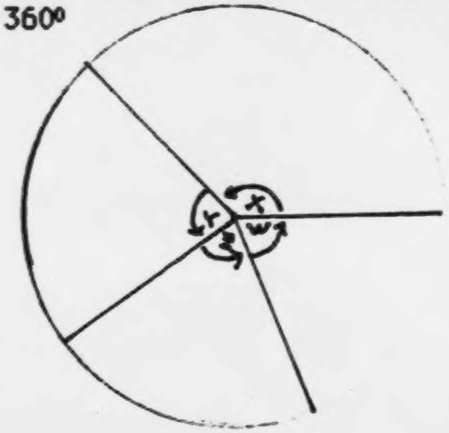
Prepare questions about these tables and graphs.

Have students suggest questions that could be answered by study of the tables and graphs.

LESSON FOUR: Making Circle Graphs

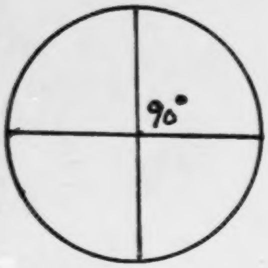
Circle graphs may necessitate dividing a circular region into sectors using angle measurement. Here are some basic facts that students must use to do this.

1. There are 360° of arc in the circle and the sum of all of the central angles of a circle is 360°

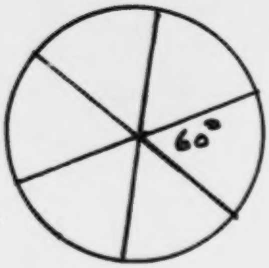


$m < x + m < y + m < z + m < w = 360^\circ$

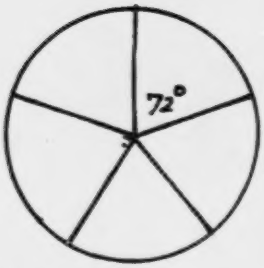
2. Central$\angle$s can be found as fractional parts of 360°. Thus the regions formed will be fractional parts of the circular region.



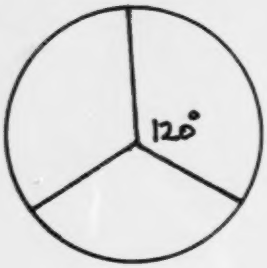
a central ∠ of 90 divides
into fourths



a central ∠ of 60°
into sixths



a central ∠ of 72°
into fifths



a central ∠ of 120° into thirds, etc.

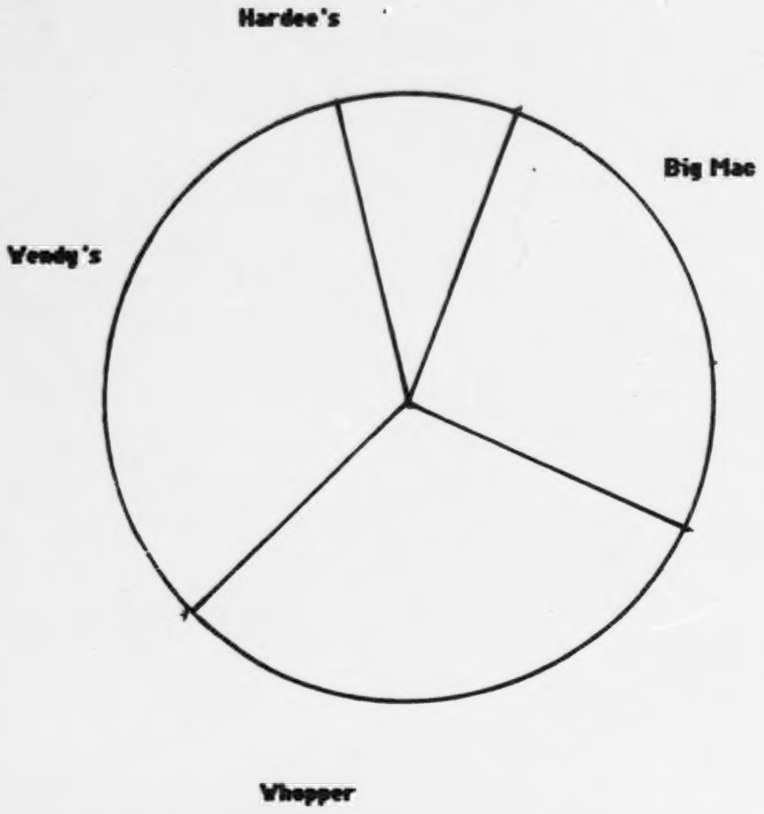
Consider this data:

	Protein Grams	Fat Grams
Big Mac - McDonald's	26	33
Whopper - Burger King	26	36
Double burger - Wendy's	44	40
Roast Beef - Hardee's	21	17

We could graph the relative amount of fat in each.

1. Total fat grams = 126
2. 360° ÷ 126 ≈ 3–° = 126
3. 3 x 33 ≈ 95° or so for Big Mac
   3 x 36 ≈ 110° for Whopper
   3 x 40 ≈ 120° for Wendy's
   Balance for Hardee's

Result:

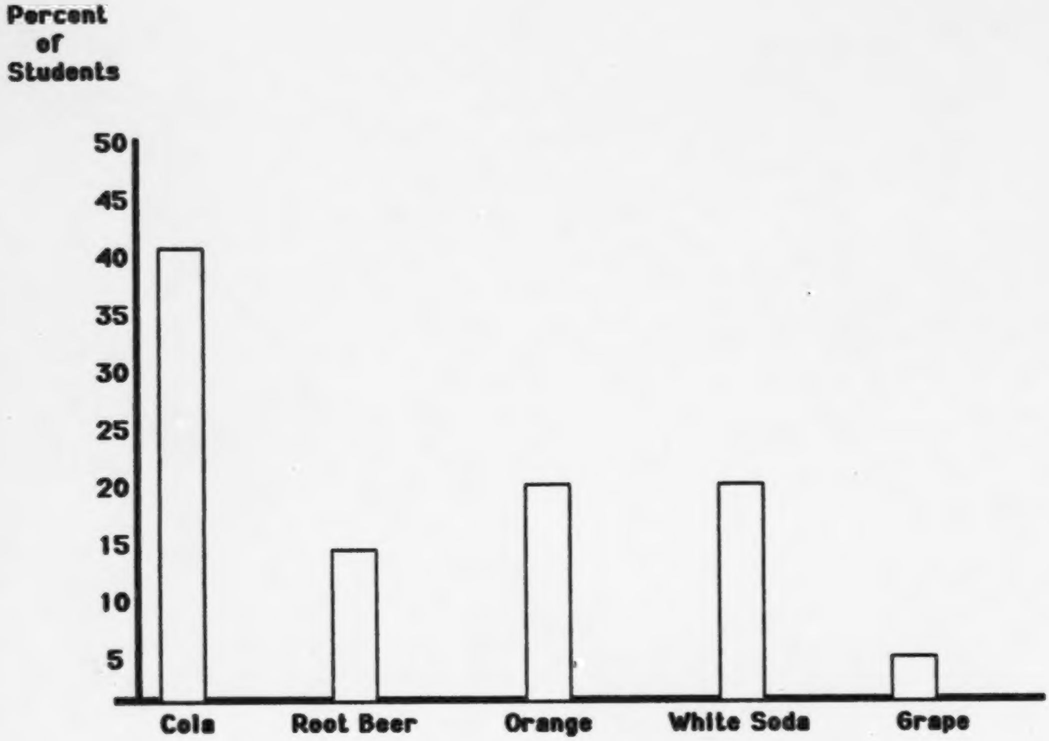


GRAPHING: Sources of Data

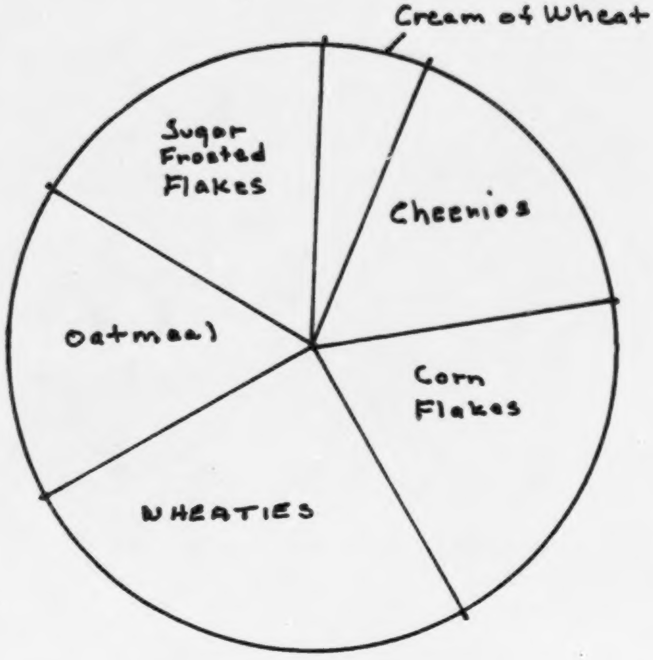
Class activity should generate data to be used for graphing activity. Some suggestions for this include:

1. "Write down a number between 1 and 10."
2. The months in which students have birthdays
3. The number of brothers and sisters
4. Target circle games - dropping onto a target circle from a height, etc.
5. Word counts from magazine and newspaper clippings.
6. Vowel counts in a set of words
7. Spinner games
8. Tossing games - into cups, etc.
9. Word lengths
10. Student heights and weights within ranges - 1 inch, 5 pounds, etc.
11. Rolling dice
12. Tossing coins - one, two, three at a time
13. Tallying days of snowfall or rainfall
14. Counting makes of cars in a parking lot

GRAPH I


Percent of Students
50
45
40
35
30
25
20
15
10
5
Cola
Root Beer
Orange
White Soda
Grape

GRAPH 2



Cream of Wheat
Sugar Frosted Flakes
Cheerios
Oatmeal
Corn Flakes
WHEATIES

TABLE 1

Day	1	2	3	4	5	6	7	
Loops Added	5	7	9	11	13	15	17	
Chain Length	5	12	16	20	24	28	32	

TABLE 2

Mileage at Start	8930	9210	9320	9510	9730
Mileage at Time Tank Filled	9120	9320	9510	9730	9900
Miles Travelled	190				
Gallons to Fill Tank	10	9.5	11	14	8
Miles Per Gallon	19				

LEVEL FIVE

MEASUREMENT

LESSON ONE: Length

Introduction: Review the measurement of length using the standard units of inches and centimeters. Use overhead transparency rulers and lines on transparencies to:

1. Measure to the nearest UNIT
2. Measure to the nearest parts of units, such as "1/4 in.", etc.

Give students the worksheets to complete. The group measurements should involve objects in the room.

LESSON TWO: Area

Introduction: Use a centimeter square graph paper with shapes traced on it to review the counting of square units within the shapes. Point out how sometimes halves of squares can be added together to get squares, while at other times good estimates have to be made of the total of several small parts of squares.

Pass out the worksheets for students to work on. They should be used in sequence:

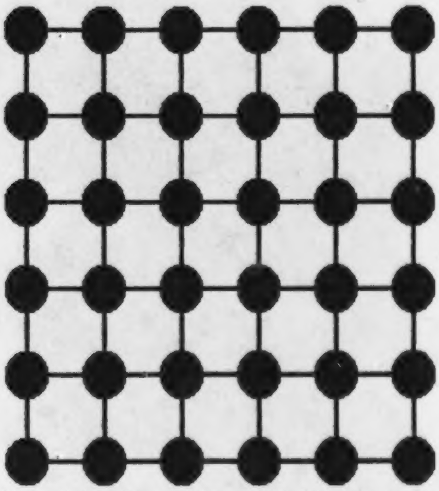
1. Shapes on graph paper

2. Shapes to cover with graph paper transparency

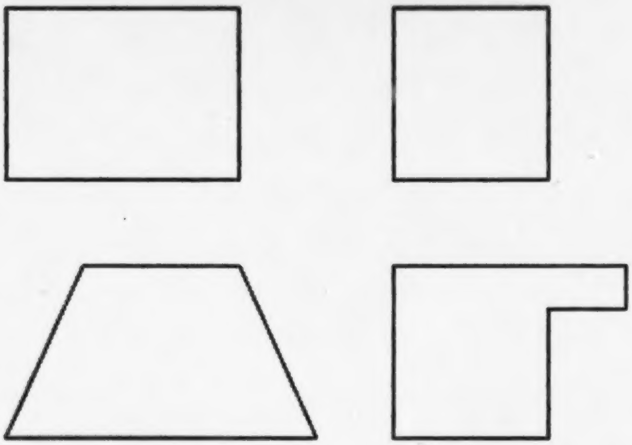
3. Shapes to find area for using dimensions given.

LESSON THREE: Geoboards

Introduction: The use of geoboards should be easy to introduce after having had graph paper introduction to area. Use geoboards that have a rectangular coordinate system as background:

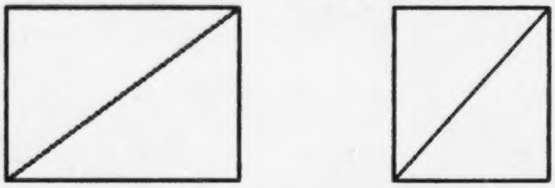


Start by framing with bands a few shapes that have right angles and sides along the lines of the coordinate system.



etc.

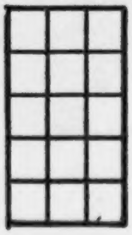
Then use a band to divide squares and rectangles into 2 parts.



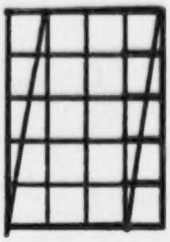
Point out that each RIGHT TRIANGLE formed has an area 1/2 that of the shape it is in.

LESSON FOUR

Make a rectangle on the geoboard:

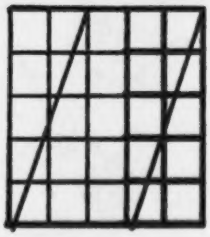


Use a second, different color band and make a parallelogram over the rectangle.



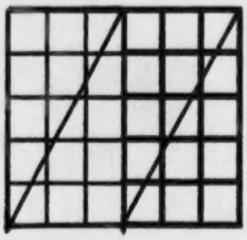
"How does the area of this shape compare with the area of the rectangle?"

Point out that the RIGHT TRIANGLE taken off is the same as that added on to the right. Each is half of a 4 x 1 rectangle. Move the second band to the new position shown:



"How does the area of this parallelogram compare with the original rectangle?

Again equal triangles are involved - one taken away and the other added to - the rectangle. Each is half of a 2 x 3 rectangle.



"What has stayed the SAME in changing the rectangle into parallelograms?" Continue questioning until you get at the idea that:

1. The base is the same, and
2. The height (distance between opposite bases) is the same.

GENERALIZATION: AS LONG AS A SIDE AND THE DISTANCE BETWEEN IT AND THE OPPOSITE SIDE ARE THE SAME, THE AREAS OF PARALLELOGRAMS WILL BE THE SAME. (including rectangles)

LESSON FIVE

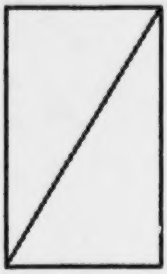
Introduction: Make a rectangle on the geoboard. Stretch a band on a diagonal to make two congruent right triangles.

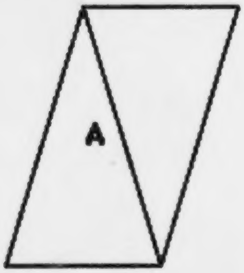
"Each triangle is 1/2 the area of the rectangle." "The rectangle and triangle have a side in common and the same height."

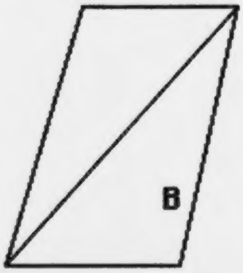
Elaborate on this theme with other rectangles. Make a parallelogram on the geoboard. Stretch a band along a diagonal to make two congruent triangles. "Each triangle is 1/2 the area of the parallelogram."

"The parallelogram and the triangle have a side in common and the same height."

If necessary, use the template to make paper rectangles and parallelograms. Have the students cut them in half along diagonals; both diagonals for parallelograms; and match them to be sure they are convinced the two areas of triangles formed are equal.





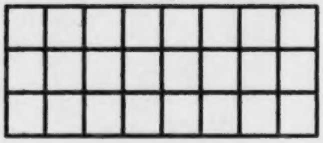


Ask why the area of triangle A is equal to the area of triangle B.

LESSON SIX: Missing Dimensions

Once students know how to use two dimensions to find area of rectangles and parallelograms, they should be able to use area and ONE dimension to find the second.

Put a rectangle made of graph paper on the overhead projector:



"What is the area inside of this rectangle?"
"What is the length?"
"What is the width?"
"If I know the length and width (point to each), how do I find the area?"
"If I know the area and one side, how do I find the other side?"

Write:

Area = 3 x 7 = 21
Width = 21 ÷ 7 = 3
Length = 21 ÷ 3 = 7

Assign the worksheets in order: (1) graph paper
(2) "open" shapes

LESSON SEVEN: Review of Volume

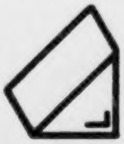
Introduction: Review the idea of CUBES in volume of rectangular solids by building a few from inch cubes or MULTILINKS. Remind the students that the volume is found using the THREE dimensions of the rectangular solid.

The worksheets provided have solids to find volume of.

LESSON EIGHT: Geoblocks

The attached forms describe all geoblocks in a set. The cube labelled "ONE" is to be used along with those circled.

Use these to show the students how volumes can be related to other volumes. In particular the triangular prisms should be shown to be half of rectangular solids like right triangles are half of rectangles. Worksheets are provided to have students calculate volumes of cubes and triangular prisms.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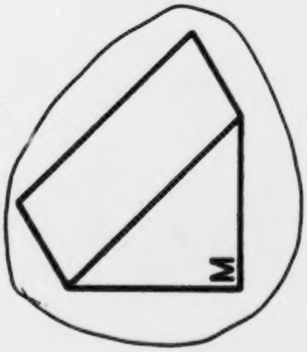
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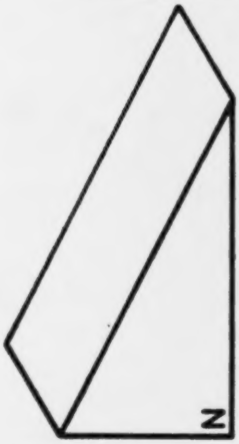
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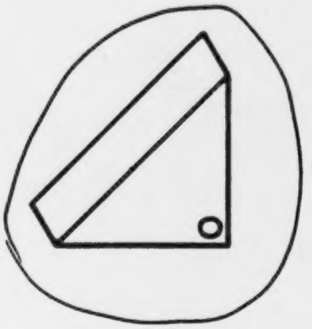
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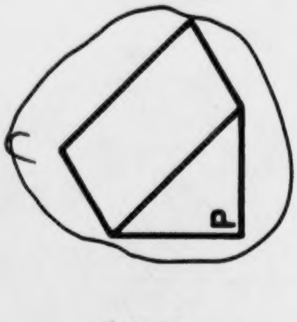
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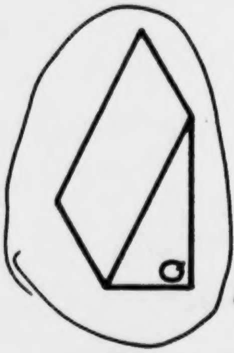
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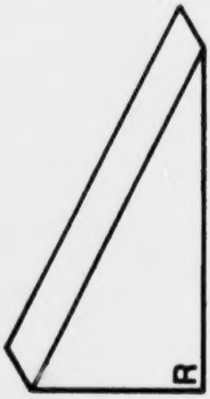
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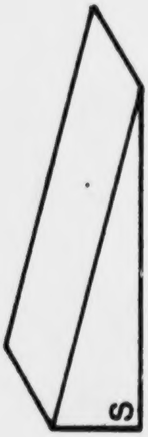
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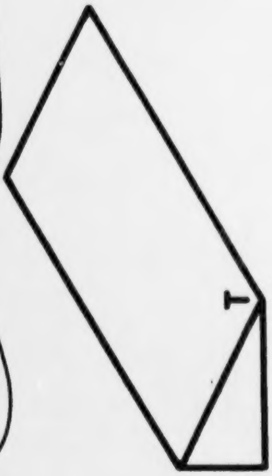
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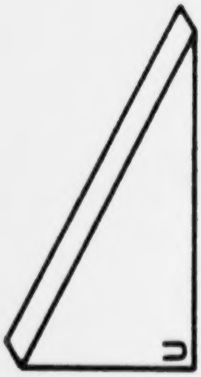
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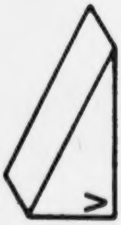
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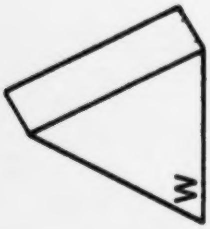
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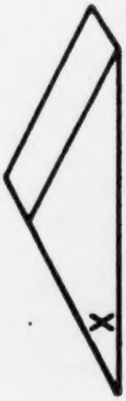
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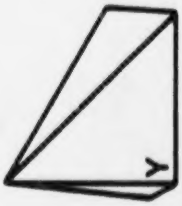
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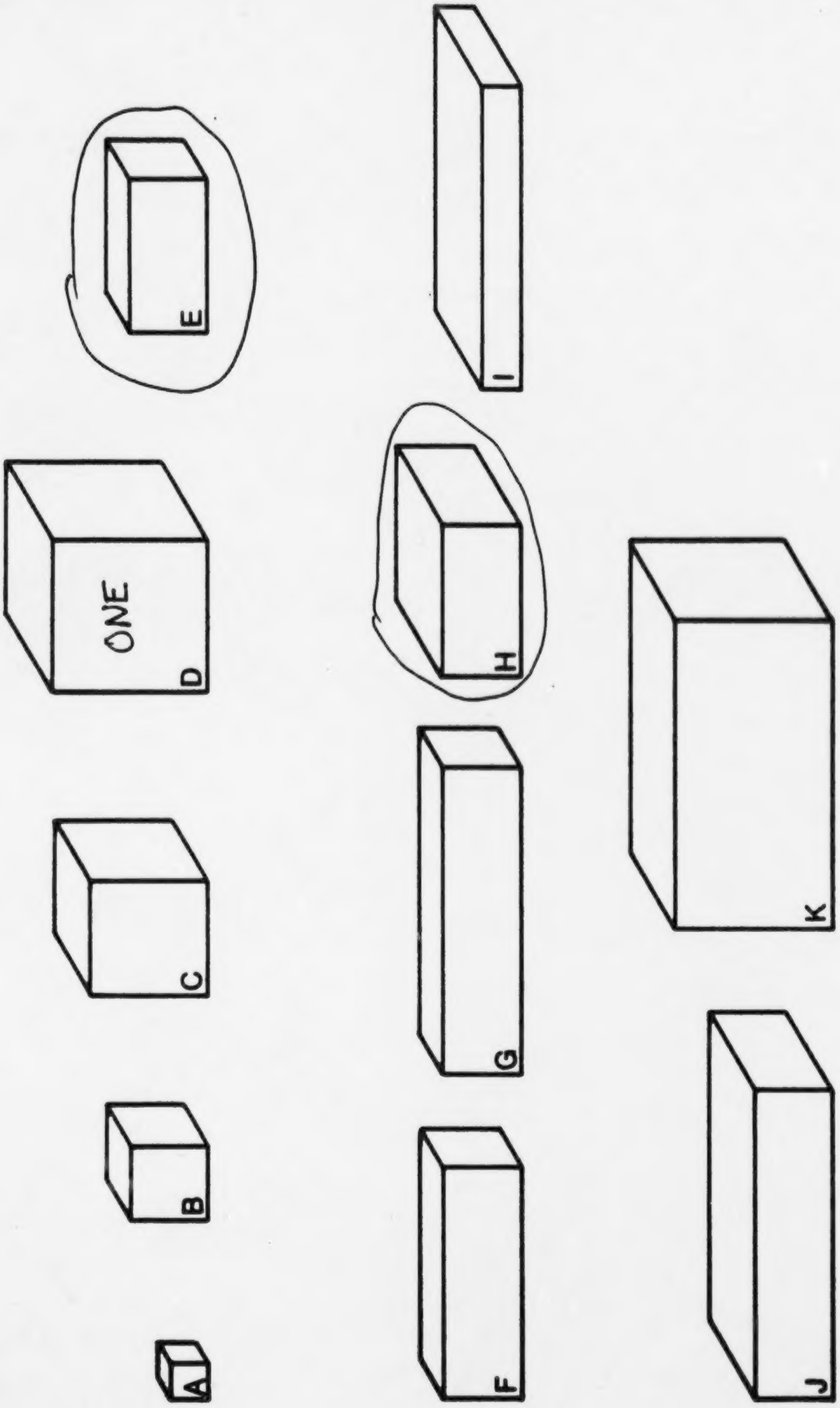

A
B
C
D
ONE
E
F
G
H
I
J
K

LESSON NINE: Creating More Problems

Write this number sentence on the chalkboard: $8 + 7 = 5 \times 3$

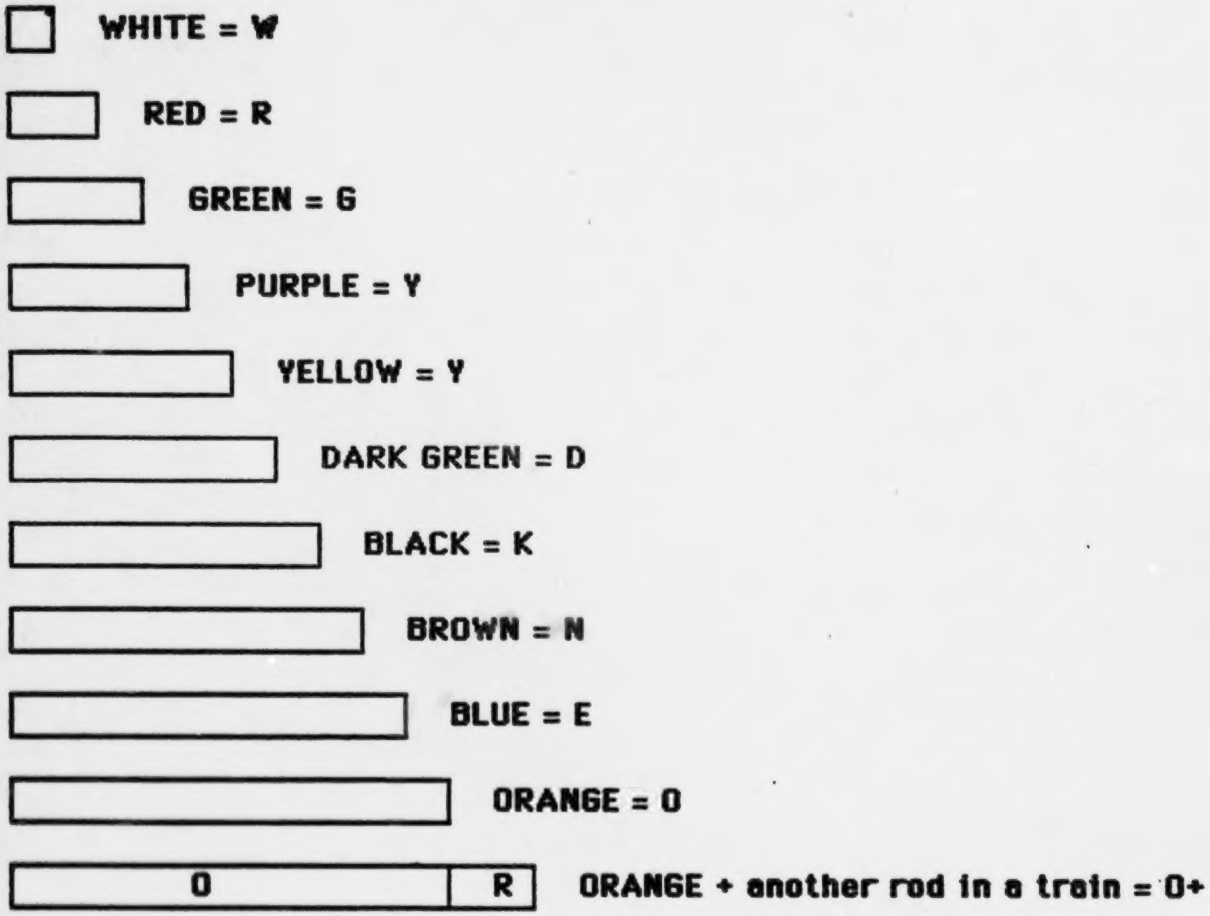
"Both sides are 15." Circle 8. $\textcircled{8} + 7 = 5 \times 3$

"Try to write a story problem that uses, 7, 5 and 3 with a question that "8" answers.

Discuss these. Pay particular attention to proper interpretation of the operations into language. Discuss the variety of problems and different kinds of problems. Follow by selecting each of the remaining 3 numbers as the "answer" and having children write these problems.

Pass out the worksheets and have children write problems.

THE ROD CODE



WHITE = W

RED = R

GREEN = G

PURPLE = Y

YELLOW = Y

DARK GREEN = D

BLACK = K

BROWN = N

BLUE = E

ORANGE = O

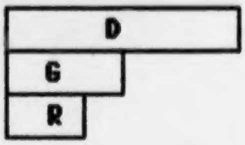
ORANGE + another rod in a train = O+

LEVEL FIVE

FRACTIONS

LESSON ONE: Review of Basic Operation

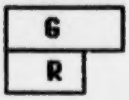
Introduction: Place a cuisenaire D rod on the overhead with G and R rods below it:



"D is one. What fraction is G? R?" "Which is longer?" "Which fraction is greater, 1/2 or 1/3?"

Join G & R.

"What is the result of adding 1/2 to 1/3?" Compare G with R:



"What is the difference between 1/2 and 1/3?"

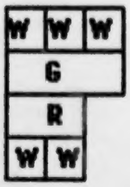
You may have to emphasize the fact the W = 1/6 when D = 1.

"How many R rods or 1/3's can be made from the G rod or 1/2?"

Write: $1/2 \div 1/3 = 1\ 1/2$.

"How many G rods or 1/2 can be made from the R rod or 1/3?"

"Only part of a G rod can be made, so the result is a fraction."



Point out the R rod is 2/3 of the G rod, so $1/3 \div 1/2 = 2/3 \div 3/6 = 2/3$.

"Is there a rod that is 1/2 of 1/3?"

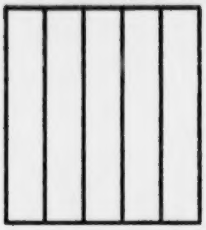
Write 1/2 x 1/3 = 1/6

"Is this same rod 1/3 of 1/2?"

Point to the three W's that are equivalent to the G or 1/2 rod if necessary.

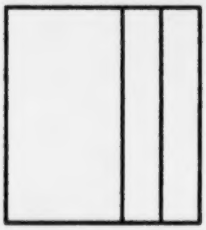
LESSON TWO: Review of a Common Denominator

Place a square divided into fifths on the overhead:



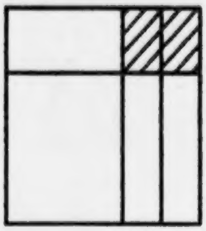
"This one is divided into FIFTHS."

Cover 3 of these:



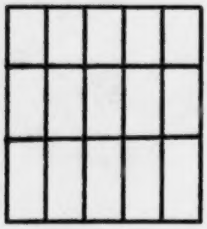
"What fraction is now showing?"

"I will take 1/3 of this fraction." Place a square divided in THIRDS over this:



"How many parts are there here?"

"What size are these parts?" Emphasize how the FIFTEENTHS are obtained.



"Does a FIFTEENTH measure a THIRD? How many are needed?" "Does a FIFTEENTH measure a FIFTH? How many are needed?"

A FIFTEENTH is a COMMON measuring unit since it measures both."

Consider this case:

3/5 , 2/3

"What is the common measuring unit?" "This is a common denominator?"

Write the fractions with this new denominator:

9/15   10/15

"Now these are like anything else that could be named, such as 9 boxes and 10 boxes, or 9 cookies and 10 cookies, so they are added, subtracted, and divided the same way."

Joining we get: $\frac{9}{15} + \frac{10}{15} = \frac{19}{15}$

Finding the difference, we get: $\frac{10}{15} - \frac{9}{15} = \frac{1}{15}$

Dividing we get: 9 ÷ 10 = 9/10, or 10 ÷ 9 = 10/9
3/5 ÷ 2/3 = 9/15 ÷ 10/15 = 9/10
2/3 ÷ 3/5 = 10/15 ÷ 9/15 = 10/9

Pass out worksheets for students to complete. Have cuisenaire rods handy for students who need to use them. Remind them to use:

D = ONE for sixths, halves and thirds
N = ONE for eighths, fourths, halves
O + R = ONE for twelfths, sixths, fourths, thirds, halves

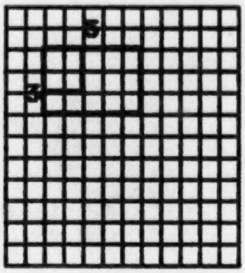
Also remind them to multiply both numerator and denominator by the denominator of the OTHER fraction in changing to equivalent fractions with a common denominator.

1/2 x 1/3
3x 1/2 = 3/6 , 2 x 1/3 = 2/6

1/5 + 2/3
3 x 1/5 = 3/15 , 5 x 2/3 = 10/15

LESSON THREE: Rectangle Multiplication

Write 2/5 x 2/3 on the board. Put a graph paper transparency on the overhead and made a 3 x 5 rectangle on it. Point out this is the product of the denominators.



"How many units are in the rectangle?"

Now make a 2 x 2 rectangle inside of this. "How many units in this?"

Our product then is 4/15.

Do a second problem as suggested by the students. Use additional worksheets that involve more complex problems.

LEVEL FIVE

DECIMALS

Background: Students must integrate the concept of fraction as "equal parts of a whole" with place value representation to understand decimal representation.

Several models can be used to illustrate decimals. Students should have been exposed to all of these at Level Four. These are:

> Cuisenaire Rods with O-the orange rod - equal to ONE. Each of the remaining rods then represent TENTHS.
>
> Base Ten blocks with the TENS pieces representing ONE. The ONES pieces then are TENTHS. When the HUNDREDS piece represents ONE, each TENS piece is a TENTH and the ONES pieces are HUNDREDTHS.
>
> Hundreds square graph paper that can be shaded in to show decimal parts.

LESSON ONE: Review: Reading decimal numerals

Write 125 on the overhead or chalkboard.

"Read this numeral in words," (one hundred twenty five - DO NOT PERMIT 'AND' AFTER THE HUNDREDS PLACE!!!)."

"What digit is in the HUNDREDS place?"

"What digit is in the TENS place?"

"What digit is in the ONES place?"

"What does the '2' count?"

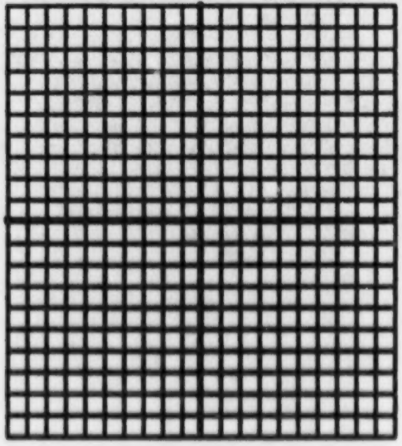
"How many tens are in the number?"

Differentiate between 12 tens and the 2 in the TENS place.

Write 3.85 on the overhead or chalkboard.

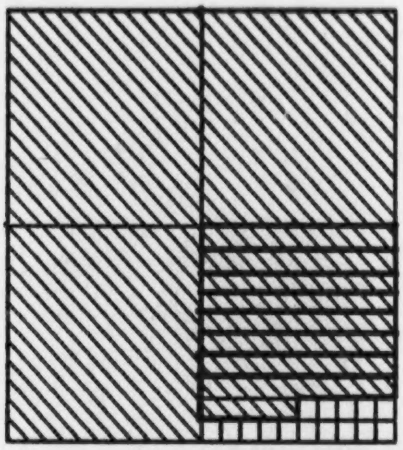
"What does the 3 count?" (ONES)
"What does the 8 count?" (TENTHS)
"What does the 5 count?" (HUNDREDTHS)

Place a hundredths graph on the overhead.



"Each square is ONE divided up into a hundred equal parts."

Shade in the 3.85 on this:



"How much is this less than four?"
"Is this more or less than half of a 4th ONE?"

Emphasize the difference between TENTHS and TENS
HUNDREDTHS and HUNDREDS

Do several examples, showing the graphic representation each time. 3.08, 2.50, 1.96 for example.

Assign the worksheets for children to complete.

COMPUTATION IN BASE TEN: BACKGROUND INFORMATION

Children should have had previous experience with oral rehearsal of multiplication facts for small numbers less than ten such as 7x8 = ?. They should also have had experience with responding to open sentence stimuli like 9x6 = ______ and ______ = 4x7 for these same numbers. The idea that multipling by the tens place so as to change each place value digit to count the next larger group size should also have been well developed and the children should have had oral practice and written practice with stimuli like 13x 10, 24x10, etc. They also should have immediate response knowledge of the effect of multiplying by the hundreds lace on which group sizes the digits in the positional notatio nrepresentation are counting, i.e., 100 x 7 = 700, 100 x 14 = 1400. Such knowledge is prerequisite to using multiplication in a place value system. The "basic facts" of place value multiplication must be done with understanding. These are before regrouping.

Counters x Counters = Counters
Ones x Base = Base
Base x Base - Base Squared
Base x $(\text{Base})^2$ = Base Cubed, etc.

In base ten these translate into seven x three = twenty-one three = 21 (after regrouping).

10 x 7 = 70 (7 tens)
30 x 20 = 600 (6 hundreds where hundred = $(\text{ten})^2$
20 x 400 = 8000 (8 thousands where thousand = $(\text{ten})^3$

Give some oral review of ten times various one and two digit numbers and one hundred times these. Then try 20 times 30. Remind students that these products have 2 important features - the counter multiplication is like that with small numbers and the place value group "size increase" feature.

LEVEL FIVE

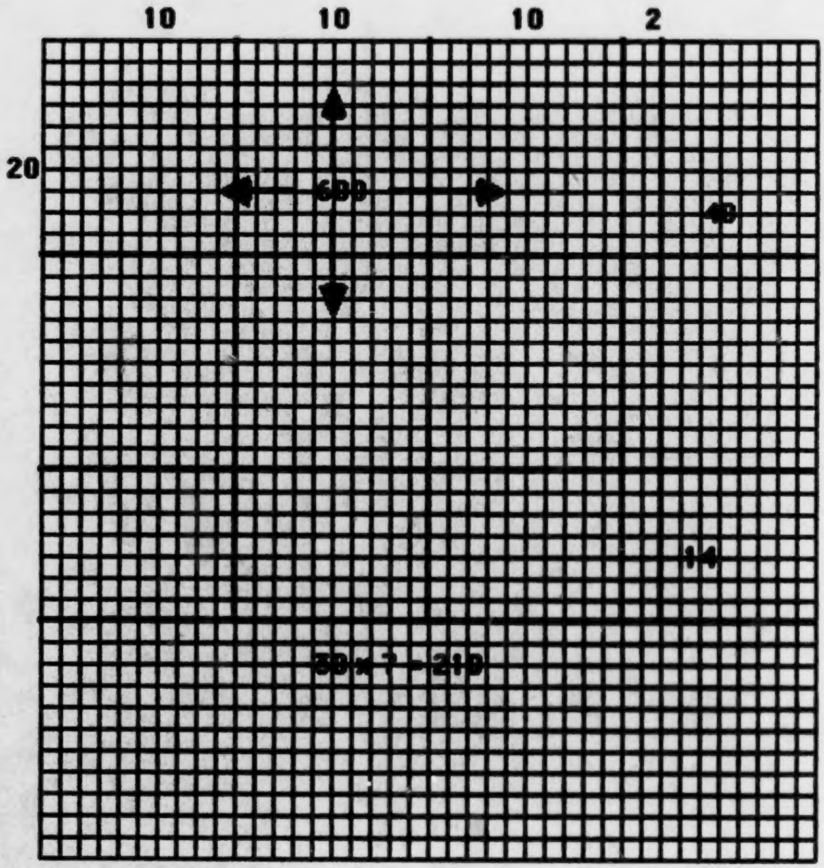
General Review of COMPUTATION:

1. Periodically give a multiplication table to complete.

2. Use overhead transparency base ten blocks. Arrange these on the overhead so a cover can gradually be slid away to reveal hundreds, tens and ones arranged in different ways. Students are to give the total shown each time the cover is moved. (see, Level Four: Lesson One)

LESSON ONE: Multiplication

Use graph paper transparency that is marked into one hundred blocks. Write a problem on the board, such as 32 x 27. On the graph paper, outline as shown:



Point out the partial products:

32
x27

32
x27

1. 30 x 20 = 600   2. 20 x = 40

32   32
x27   x27

3. 7 x 30 = 210   2 x 7 = 14
total = 864

Write this as expanded notation:

32 = 30 + 2
27 = 20 + 7

Show the same 4 partial products

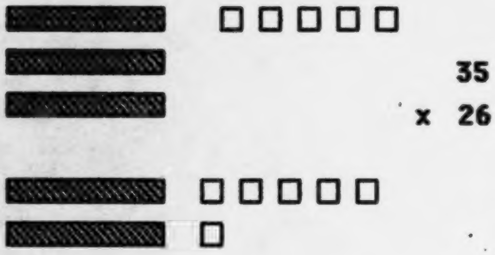
Write this as a binomial product: (30 + 2) (20 + 7) and multiply first by 30, then by 2 to get 600 + 210 + 40 + 14 = 864

Remind the students T can represent ten so 30 = 3T, 50 = 5T
$T \times T = T^2$, so $3T \times 5T = 15T^2$ or 15 x 100 = 1500

Pass out worksheets for students to work on. Allow for use of base ten blocks if these are needed by some students.

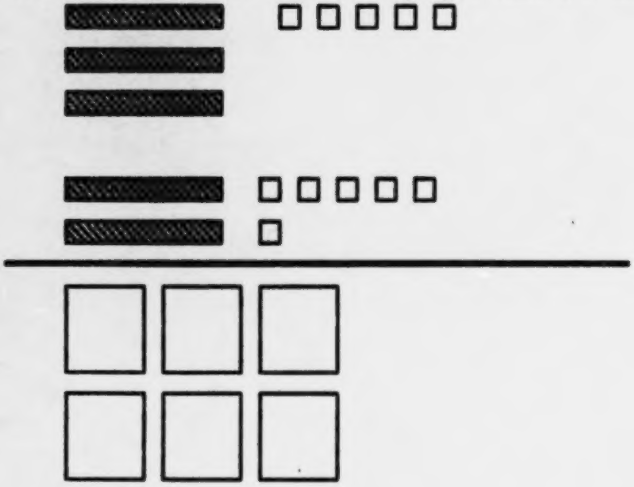
LESSON TWO: Multiplication 2

Use base ten blocks in overhead transparency form. Write a multiplication example and place the corresponding base ten blocks in standard computation form:

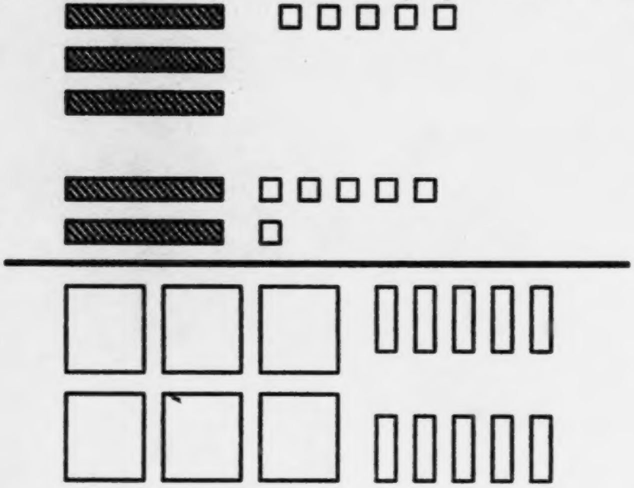


35
x 26

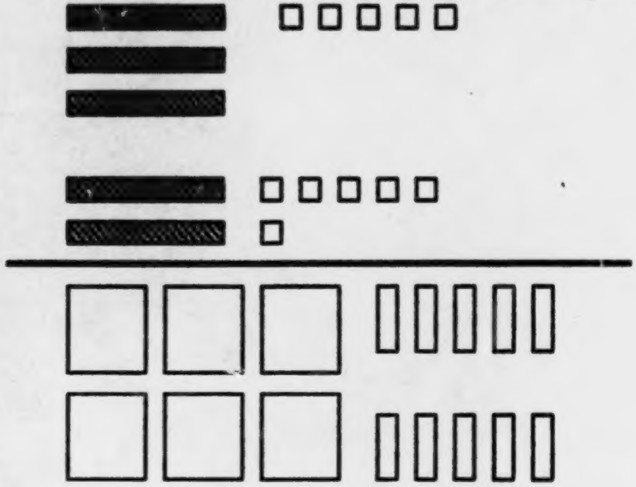
"What is the result of multiplying 2 tens x 3 tens?" Put the hundreds in:



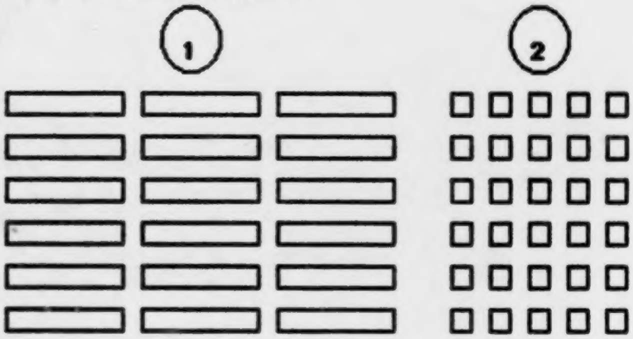
"What is the result of multiplying 2 TENS x 5 ONES?" Put in the tens



"What is the result of multiplying 6 ONES by 3 TENS?"



Put these tens in place:

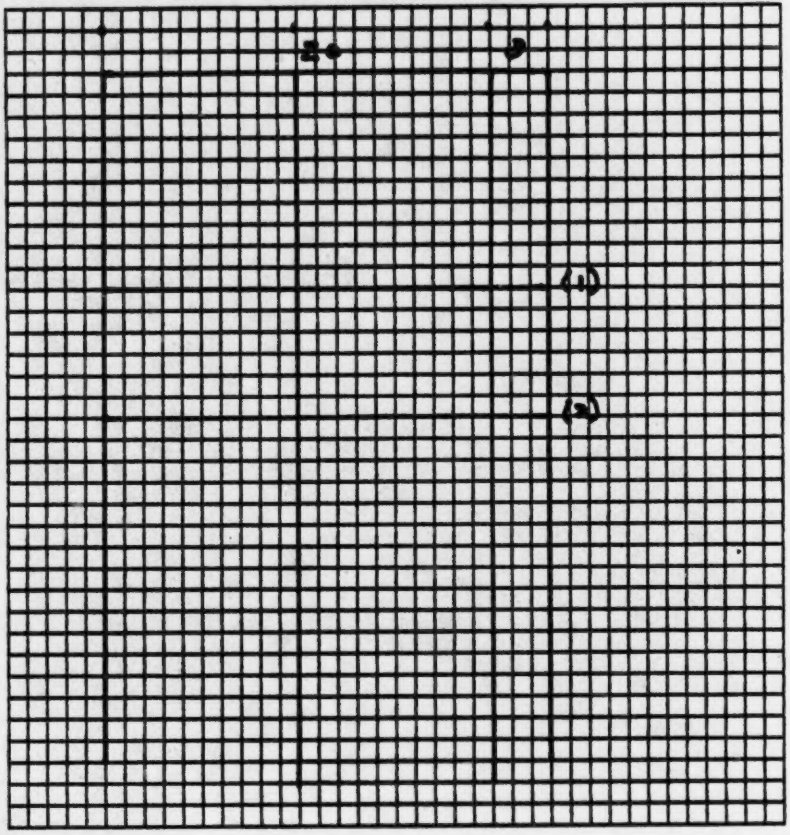


"What is the result of multiplying 6 ONES x 5 ONES?"
Put these in also (2)
"What exchanges can be made?
First 30 ONES for 3 TENS, leaving 0 ONES
Then 20 TENS for 2 HUNDREDS, leaving 1 TEN
Then 10 TENS for 1 HUNDRED
The total is then 9 HUNDREDS
1 TEN
0 ONES

Do a second example. Then pass out worksheets to do. Have base ten blocks available for those who need them.

LESSON THREE: Division

Use overhead transparency graph paper. Write a division example such as: 385 ÷ 23.
Mark off the division on the graph paper:



"We must find the other side of the largest rectangle we can make from 385 units area."

Go down ten units and draw a line (1). "How many units are used up if the other side is here?" (230)

Subtract this from:

$$\begin{array}{r} 385 \\ \underline{-230} \\ 155 \end{array}$$

"We still have 155 units. How many 23's are in this?"

This is a good chance to have the students estimate. Encourage them to use a low estimate.

"We'll try 6 units." (2)

"This uses how many more units?"

Subtract 238 from 155:

$$\begin{array}{r} 155 \\ \underline{-138} \\ 17 \end{array}$$

"17 is too small to make another row. So our longest side is 16. 17 is a remainder."

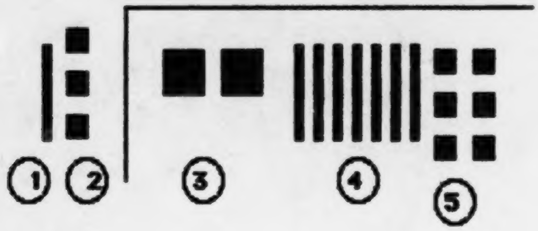
Write: 385 ÷ 23 = 16 R 17

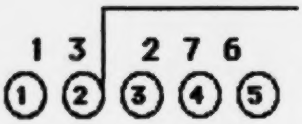
Do a second example. Pass out worksheets for students to complete.

LESSON FOUR: Division by Places

Introduction: This is a review of the traditional long division algorithm, but with an emphasis on the place value nature of the operation. Write the following on the board: 276 ÷ 13.

Arrange the following on the overhead projector with base ten blocks. Relate each group of blocks to the digits in the standard form:

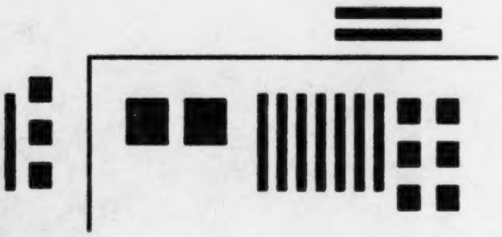




"What place is the result of dividing hundreds by the tens?" (TENS)

"How many tens result from 2 hundreds ÷ by 1 ten?" (2)

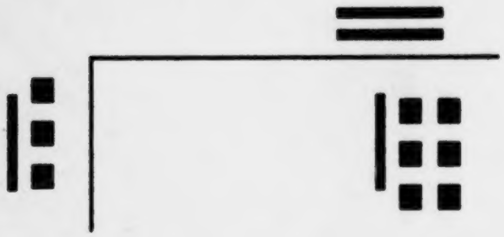
Put the two tens as shown:



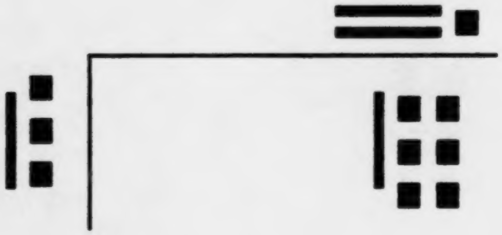
"2 tens x 1 ten equals how many hundreds?" (2)

"2 tens x 3 ones equals how many tens?" (6)

"We have used 2 hundreds and 6 tens." Remove these.



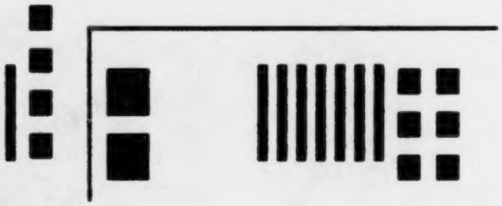
"What is one ten divided by 1 ten? (1 unit). Put that in the quotient.



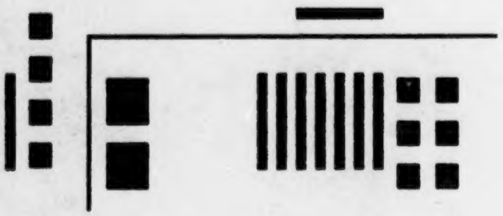
"This unit times the divisor uses 1 ten and 3 ones." Remove these.

"These 3 units are the remainder. Write: 276 ÷ 13 = 21 R 3.

"What if the divisor is 14?"

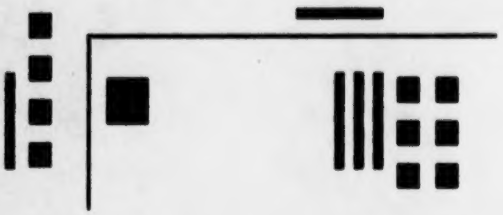


"2 hundreds divided by 1 ten still gives an ESTIMATE of 2 items. But when we multiply 2 tens x 4 ones, 8 tens would result, and we have only 7 tens so the ESTIMATE is too high. We can use only 1 ten.



"1 ten multiplied by 1 ten gives 1 hundred." Remove one.

"The ten multiplied by 4 ones gives 4 tens." Remove 4.

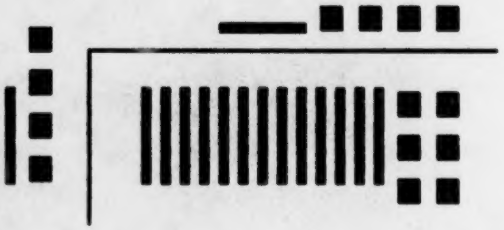


"We have already divided hundreds by tens, so what must we do with the hundred?" (trade for ten tens.)

"Then we can try numbers of one 9 or less."

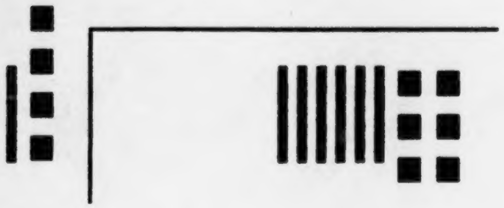
"Why must these be 9 or less?"

"Considering the number of ones in the divisor, what number less than 9 could you be sure of, so that what is left of the dividend is enough?" Try 5.



"5 times one ten = 5 tens." Remove 5 tens.

"5 times 4 ones = 20 ones = 2 tens. Remove 2 tens.



"That was a low estimate. No harm is done. We can still make the quotient larger. How much more should we make it?" (accept any suggestion of 4 or less and try it.) The result should be 276 ÷ 14 = 19 R 10.

Contrast this with:

	10	4
10	100	40
5	50	20
4	40	16

276
140

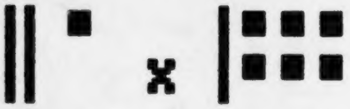
136
 70

 66
 56
---
10 remainder

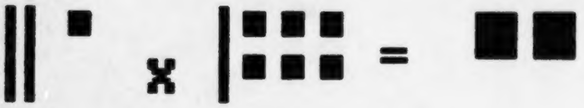
Have students work in pairs on worksheets provided.

LESSON FIVE: Horizontal Form

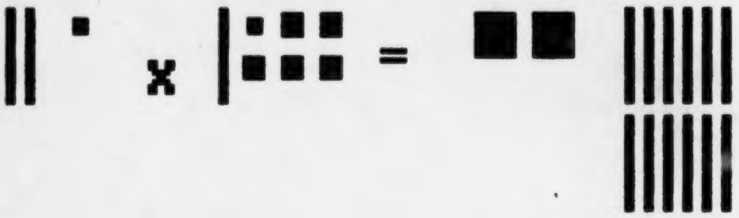
Introduction: Write 21 x 16. Arrange base ten blocks on the overhead.



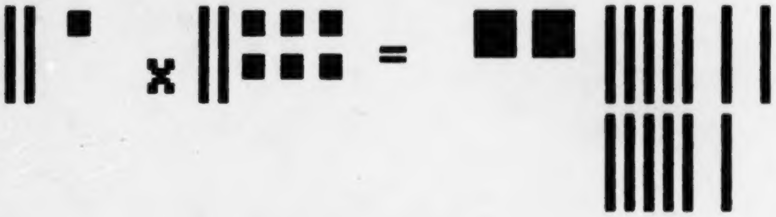
"2 TENS x 1 TEN equals how many hundreds?"



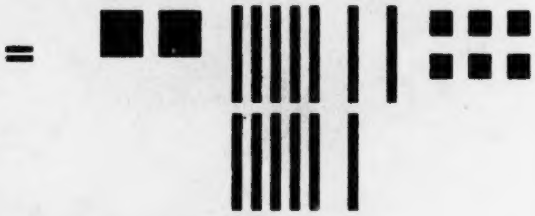
"2 tens x 6 ones equals how many tens?"



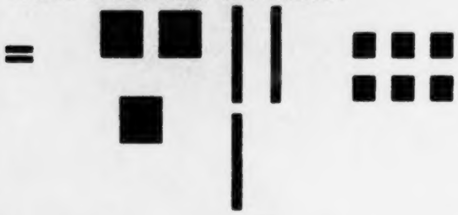
"One x one ten equals how many tens?"



"One equals 6 ones equals how many ones?"



"What trade can we make?"



Show this in symbols.
(20 + 1) (10 + 6) = 200
(20 + 1) (10 + 6) = 200 + 120
(20 + 1) (10 + 6) = 200 + 120 + 10
(20 + 1) (10 + 6) = 200 + 120 + 10 + 6 = 336

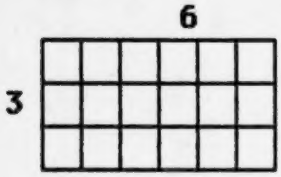
Have students work in pairs on worksheets as provided.

LEVEL FIVE

GEOMETRY

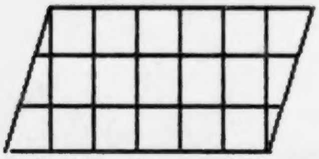
LESSON ONE: Review Area

INTRODUCTION: In Level Four, students had to find areas using geoboards, tangrams and pattern blocks. This lesson is to review these ideas. Use a graph paper rectangle on the overhead:

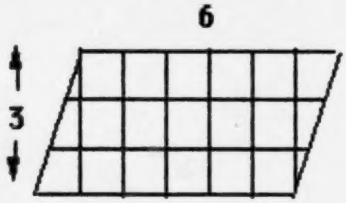


"The area in square units in this shape is the product of 3 x 6 = 18.

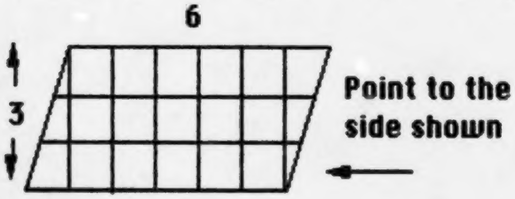
Alongside this place the following graph paper parallelogram.



"What is the area of this shape?"
Write in the dimensions as shown:



"The areas of rectangles and parallelograms are found by multiplying the height and a base.



Point to the side shown

"Is this the height?" Discuss the difference between the height and the "slant" side of the parallelogram. Pass out the worksheets for students to complete.

LESSON TWO: Review Perimeter

INTRODUCTION: Students have had experience finding perimeters in Level Four. This lesson is to review that concept.

"Draw a shape that has a lot of perimeter, but a little area."

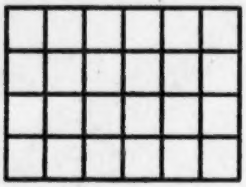
Discuss these to emphasize perimeter is length of lines "around" the shape - are is space "within" the shape.

"Draw a shape that has a lot of area, but little perimeter." The shape that encloses the greatest area with the lest perimeter is the circle. Discussion of this question should eventually lead to that idea. Remind the students that triangles have 3 sides to add together to get the perimeter, rectangles, squares and parallelograms have 4 sides to add together to get the perimeter, etc.

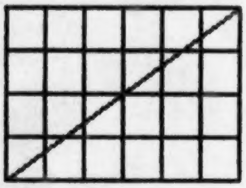
Have pairs of students complete the perimeter worksheets.

LESSON THREE: Triangles

Introduction: Put a graph paper transparency rectangle on the overhead projector.:



"What is the area of this rectangle?" Draw a diagonal in this:



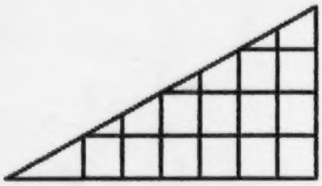
"How many triangles are found?"

"Each has what part of the area of the rectangle?"

"Can any rectangle be divided into two parts this way?"

"Will the two triangles formed always each be one half of the rectangle?"

"Place a graph paper triangle transparency on the overhead."



"What is the height of this?"

"What is the long side of this?"

"What is the area of this?"

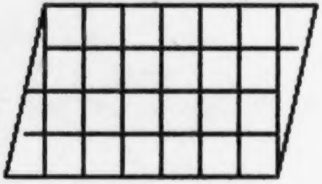
Remind the students this is half of a 4 x 7 rectangle so that the area is 14.

"Count and combine the squares to find the area a second way."

Have students work in pairs on the worksheets provided.

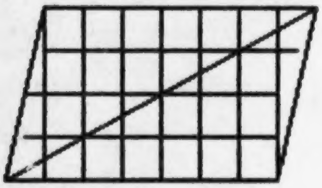
LESSON FOUR: Other Triangles

Introduction: Put a graph paper transparency parallelogram on the overhead.:



"What is the area of this parallelogram? It is 4 high and 6 on the base."

Draw a line in connecting two opposite vertices:



"Are the two triangles formed alike?"

"Each is what part of the parallelogram?"

"What is the height of the parallelogram?"

"What is the height of each triangle?"

"What is the base of the parallelogram?"

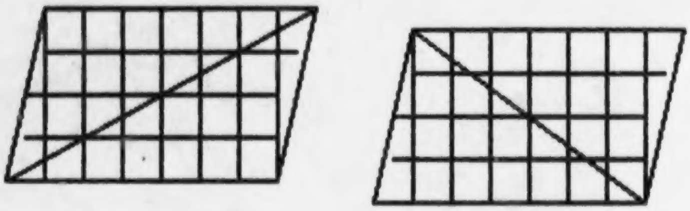
"What is the area of the parallelogram?"

"What is the area of each triangle?"

In summary:

Parallelogram 4(ht.) x 6 (base) = 24 area. Triangle is 1/2 of this or 12 each.

Use a second graph paper parallelogram just like the first. Draw the other diagonal on it:



"Are the two triangles in the second parallelogram alike?"

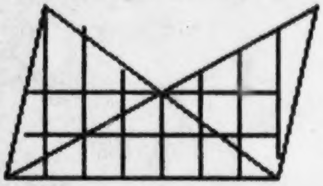
"Is each half of the parallelogram?"

"Does each have the same area as the triangles on the other parallelogram?"

"Does the area in one triangle in the first equal the area of one triangle in the second?"

"Do all four triangles have the same area?"

Discuss this thoroughly. The triangles ALL have the same height and the same "base." Consider two triangles - one from each parallelogram by cutting the transparency and placing one triangle on top of the other:



"These have the SAME base (point to it - 6) and the SAME height (point to these - 4), so have the same area." Each is half of the SAME parallelogram."

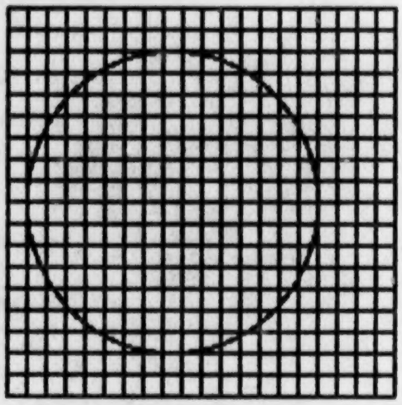
LESSON FIVE: Circles circumference

Introduction: This will be a new topic for the students. Materials necessary are solid disks of different sizes, plastic covers of cans, empty cans, such as coffee cans, party nut containers, etc.

Have the students use string and measure the distance "AROUND" (C□ several cans, disks, etc. Also have them measure the distance ACROSS (d) the widest part of the circle. The results can be recorded in the accompanying record form.
The results of calculating the ratio of C/d will be somewhere in the range 3 - 3 1/4 or 3 - 3.2. The idea to emphasize is that it is CONSTANT, no matter how large the circle.

LESSON SIX: Circles - Area

Introduction: Have students use the squares on the graph paper transparency to estimate the AREA of the circle.



"What is the distance Across (DIAMETER) of the circle?"
"The CIRCUMFERENCE (distance AROUND) is a little more than 3 x this."

"Find CD divided by A". This should be near 4. Show a second circle and have the students find A, C,d and CD/A. This should be near 4.

Students should complete the table on the worksheet using transparency graph paper to lay over circles of different sizes.

The shape that encloses the greatest area with the least perimeter is the circle. Discussion of this question should eventually lead to that idea. Remind the students that triangles have 3 sides to add together to get the perimeter; rectangles, squares and parallelograms have 4 sides to add together to get the perimeter, etc. Have pairs of students complete the perimeter worksheets.

LEVEL FIVE

VOLUME: GEOBLOCKS

Background: Review Level Four activities with geoblocks. The geoblocks in the original non-metric ESS set are described in the accompanying sheets. There are 7 different, but related, rectangular prisms. There are four, but different, cubic rectangular prisms. There are 13 triangular prisms that are related to the rectangular prisms. There is one right pyramid that is related to one of the cubes. There are enough of each kind of block in the full set so that the activities in the lessons can be done at a geoblock center. In this way only one set is needed for the class. The important ideas for students to get are:

1. Solids that are the same fractional part of the same larger solid have the same volume

2. Solids that are the same multiples of a given smaller solid are equal in volume

3. Solid volumes are usually measured by cubes

4. Two or more shapes with the same volumes need not have the same surface areas

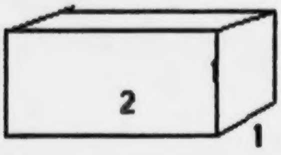
5. Triangular prisms are always half of some parallelepiped just as triangles are half of some parallelograms. Right triangular prisms are half of some rectangular prisms, just as right triangles are half of some rectangles.

6. Pyramids are always 1/3 of a prism with the same base.

The worksheets include worksheets for center activities.

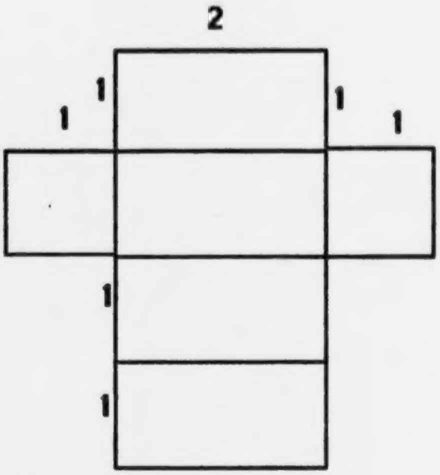
"Geojackets" are made by tracing ALL of the faces of a solid in a net so that, if cut out of paper or cardboard, the net can be folded to make a hollow facsimile of the solid. An example is:

Solid



Volume 1 x 1 x 2 = 2

Geojacket



Area = 1 + 1 + 4(2) = 10

Students should have transparent graph paper (1/2 in. squares) available to find total surfaces of blocks shown by geojackets.

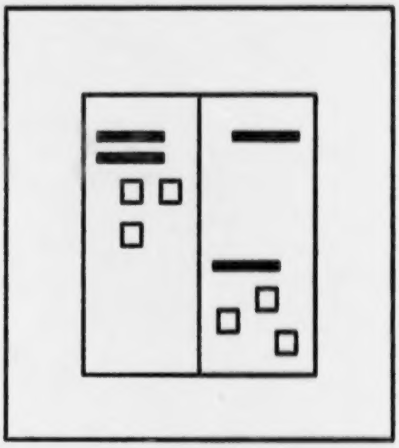
LEVEL FIVE

RELATIONSHIP

Students at this level should have a fairly well developed sense of equality, so a little review of this is all that is needed. However, they do need further developmental work with inequality.

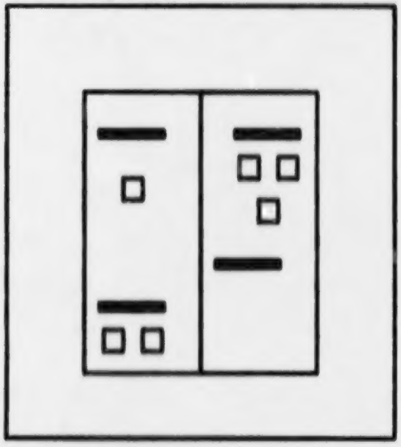
LESSON ONE:

Use base ten blocks and a split board on the overhead. Place an arrangement as shown:



"A number sentence that shows this is: 23 = 10 + 13."

Rearrange to:

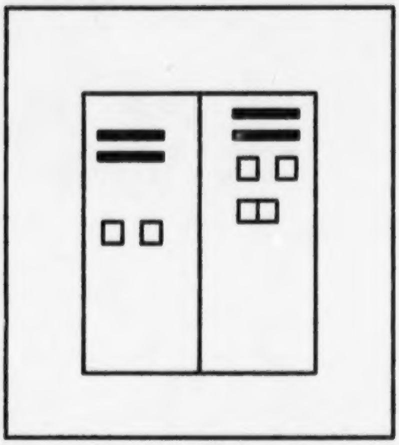


"Write a number sentence to show this." (11 + 12 = 10 + 13)

Do a few more and then have the children work on the worksheets. Provide base ten blocks for those who seem to need them.

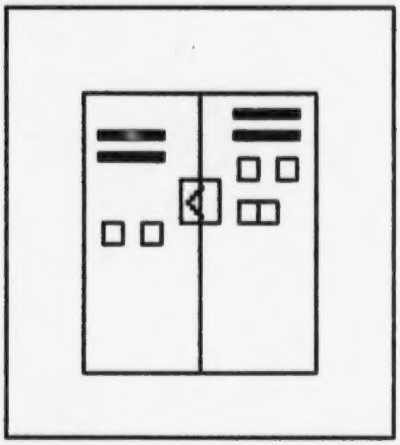
LESSON TWO:

Prepare overhead transparency inequality signs to use with a transparency split board. Place the following arrangement of base ten blocks on the overhead:



Show the inequality sign. "Which way should this sign point?"

Place it:



Remind the children it always points to the smaller number. Do two or three more and then have the children work on the worksheets provided.

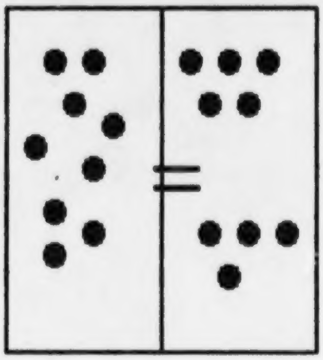
LEVEL FIVE

OPEN SENTENCES: CONCRETE

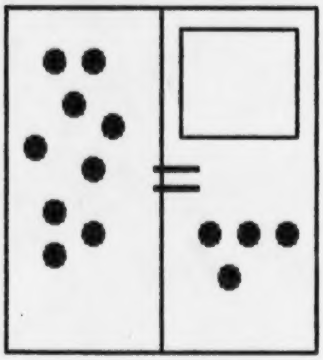
Introduction:

LESSON ONE:

Students had work with open sentences involving the use of Cuisenaire Rods at Level Four. This is a different concrete model. Put the folowing split board arrangement on the overhead projector:



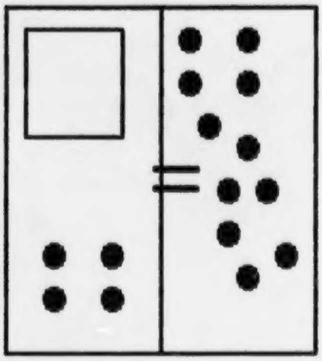
"How many are on each side?" Write: 9 = 9. Cover up the top five.



"We could write this as: 9 = ☐ + 4."

"Because we know 9 = 5 + 4, we know five is under the covered part or 5 goes in the ☐ ."

Put the following on the overhead:



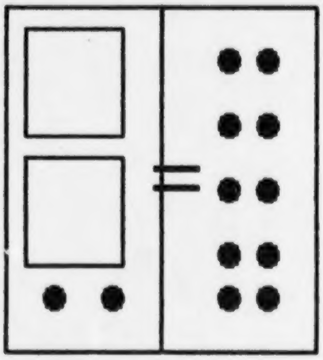
"How many chips are hidden in this case?" Write:

$$\square + 4 = 11$$

$$\square + 4 = 11$$

"Seven is the missing number and goes in the $\square$."

Put the following on the overhead:



"There is an equal number under each cover: How many are under each?"
Write:

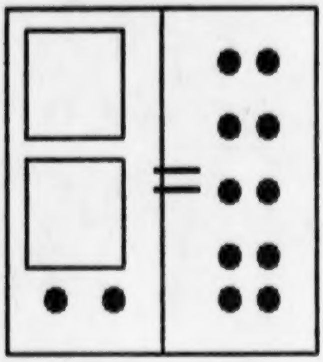
$$2\square + 2 = 10$$

$$2\square + 2 = 10$$

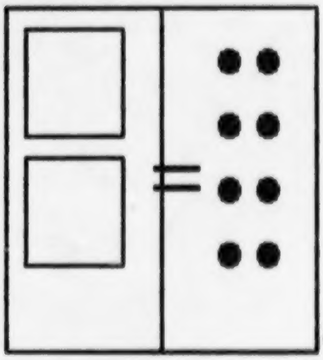
Write:

$$\begin{array}{r}7 = 7\\ \underline{-2\ -2}\\ 5 = 5\end{array}$$

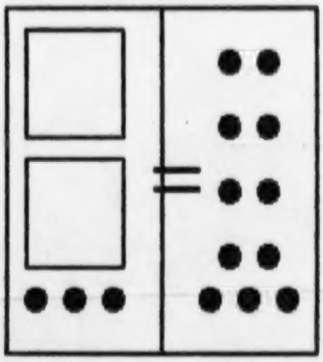
"As long as we SUBTRACT the same amount from both sides of an equality, it remains an equality." "Look at the last problem."



"Would it be easier to see what is under each if 2 chips were taken away from each side?"



Put the following on the overhead:



"How many chips should we remove from each side?" "How any chips are under each cover?"

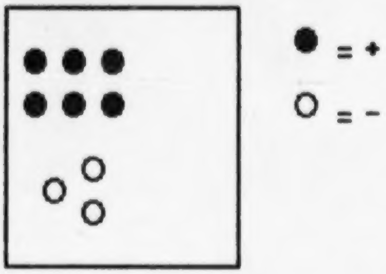
Have pairs of students complete the worksheets provided.

LESSON TWO: Review of Signed Numbers

Introduction: This topic was introduced at Level four. The important idea to reinforce is:

To Make More Positive - add +, subtract -
To Make More Negative - add -, subtract +

Put the following array of 2 different color chips on the overhead. Define one color as =, the other color as -.

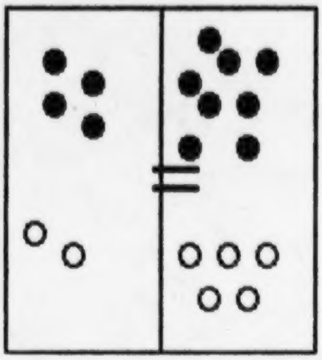


"What number does this show?" (+3)
"What should I add to make it more +?"
Use a few cases 2 more +, 3 more +, to get +8, etc.

"What could I subtract to make it more +?"
"What would I have to do to subtract 4 negatives?"

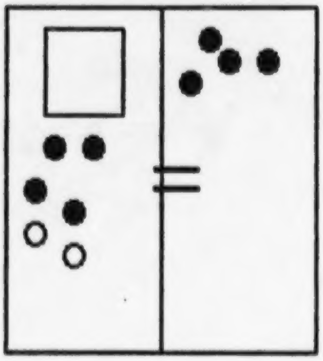
the idea of changing to a new form by simultaneously adding or subtracting EQUAL numbers of + and - should be reinforced here.

Put this on the overhead:



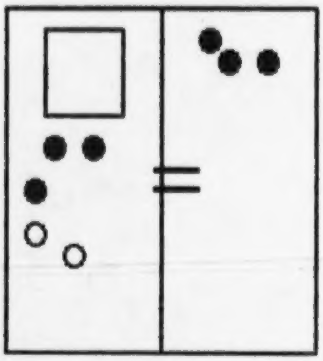
"Do these show the same number?" "What is the number?" (⁻3)

Consider this:

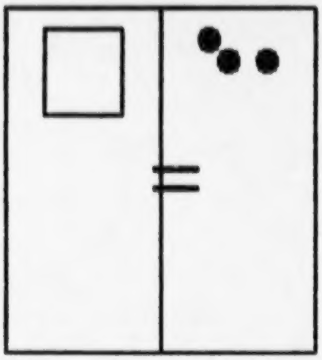


"What do you think is under the □?" "Why do you think that?"

Remove a ● from each side:

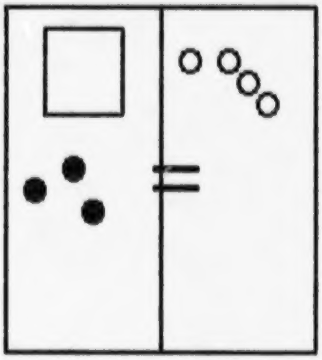


"Do the + and - on the left side add to 0?" "Then we can remove them."



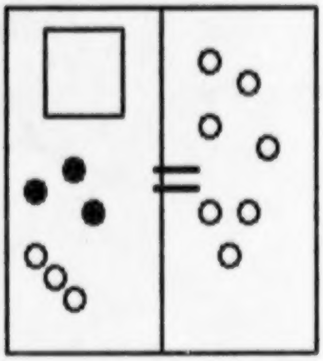
"The board now shows ☐ = +3.

Show this:

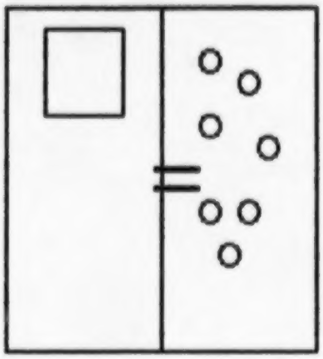


"What would you add to each side so the left side would be

+ 0?". Do that:



"What does the board simplify to?"



"So: □ = $^{-}7$."

Remind them that the board stays in balance if the same thing is added to or subtracted from each side.

Have students work in pairs on the worksheet using chips and split boards if they need these. Remind them the goal is to get

□ + 0 on one side.

LESSON THREE: Numerals

Introduction: Students should work with 2 different color chips and split boards until they clearly see:

1. The same number of + chips and − chips add up to 0.
2. □ + 0 =

3. The same number of + or - can be ADDED TO or SUBTRACTED FROM BOTH SIDES of the equality without changing it.

Then the symbols can be worked on. Write the following on the board or overhead projector:

$$\square + 4 = 7$$

"How much must be removed from the left side to get the

alone, or $\square + 0$?" Subtracting 4 from BOTH SIDES gives:

$$\square = 7 - 4 = 3$$

Put the following on the board or overhead: $2 = \square - 5$.

"What must we do to get $\square + 0$ on the right side?" (add 5)

"Adding 5 to BOTH SIDES gives: $5 + 2 = \square$

$$7 = \square$$

Write: $2\square + 1 = 9$

"What must be true of the right side to find the number for $\square$?"Work on the idea $2\square$ means the other side is a multiple of 2 or divisible by 2.

"Do we add to BOTH SIDES or subtract from BOTH SIDES to get 2 + 0?"

This gives: $2\square = 9 - 1 = 8$, so $\square = 4$.

Do: $2\square - 3 = 7$

1. The right side must be divisible by 2
2. Something is ADDED to BOTH SIDES to get 2 + 0 =

Complete it: $2\square = 7 + 3 = 10$

$$\square = 5$$

Do as many as needed to translate from the concrete models to doing the same operations with symbols. Have students work on the worksheets in pairs. Have split boards and 2 colors of chips for those who seem to need that support.

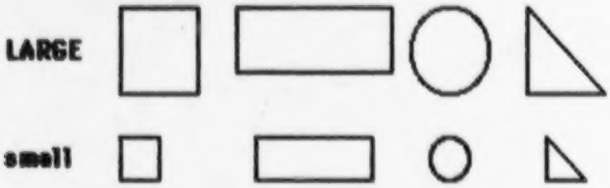
LEVEL FIVE

LOGIC

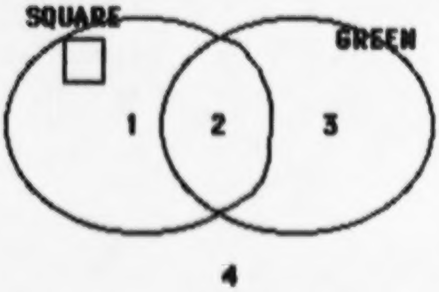
Background: Students will have had experience with recognition of similarities and differences of geometric shapes. They have also had an introduction to AND, NOT and IF-THEN. At this level, these are to be reviewed and OR is to be introduced. It will be helpful to review the Level Four lessons on this topic.

LESSON ONE: Review of AND

Introduction: Use a full set of transparency attribute pieces. Make these of red, green and blue transparency film using the shape template provided. A full set should be:



This is a total of 24 pieces. Place the following arrangement on the overhead:



One at a time show a piece and ask the students whether it should go into 1, 2, 3 or 4.

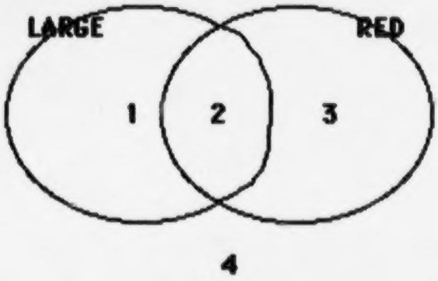
After all pieces have been sorted, ask the students how to describe the pieces in 2. (GREEN AND SQUARE)

"How would you describe the pieces in 1? in 3? in 4?"

Give plenty of time for discussion of these. Some possibilities are:

for 1 - all squares that are not green
    red and blue squares
for 3 - all blue pieces that are not squares
    blue rectangles, triangles and circles
for 4 - red and green that are not squares
    red and green rectangles, triangles and circles

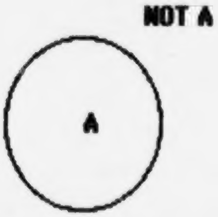
Do another:



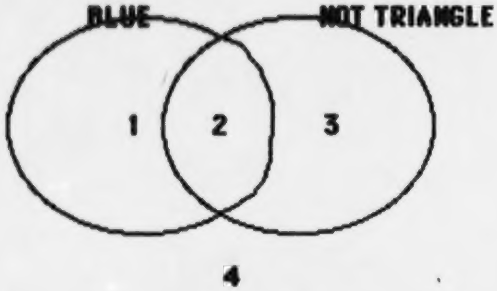
Assign pairs of students worksheets to complete. Each pair should have a set of the logic materials to use.

LESSON TWO: Use of NOT

Introduction: Using NOT alone to show the absence of a property for attribute is not too difficult for students. They recognize when we have this situation.:

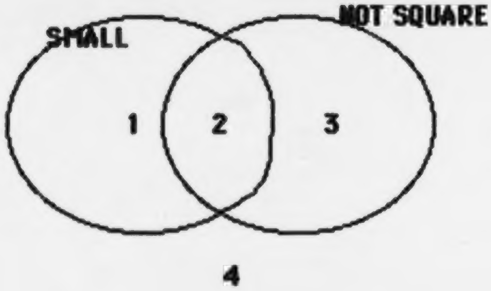


It is the use of NOT combined with AND that presents problems to many. Use a transparent 2 loop workmat and transparent attribute pieces. Label the two loops:



Pick up pieces one at a time and ask where the piece goes - in region 1, 2, 3, or 4. Emphasize why the triangles must go in 1 or 4 as these are placed. Continue until all pieces are sorted.

Arrange this situation:



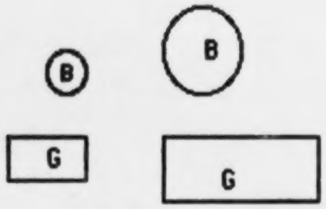
Have all pieces showing on the overhead.

Ask students which pieces should go into region 1. Discuss this as to the appropriateness of the pieces suggested. Do the same for each of the regions 2, 3 and 4 until all pieces have been sorted properly.

Assign pairs of students to work on the worksheets. Have the attribute pieces available for them to use.

LESSON THREE: Use of IF-THEN:

Introduction: Place a small collection of the attribute blocks on the overhead. An example is shown to illustrate the kinds of questions to ask the students:



"If I see a rectangle, what color is it?"

"I can say 'If rectangle then green...'."

Write: If ▭ ,then G

"If a piece is blue, what shape must it be?" Write:

If B, then ○

"Suggest other IF-THEN sentences that are TRUE of this set of pieces." Write down all suggestions. The complete list is then:

If G, then c

If ○ , then B

If ▭ , then G

if B, then ○

Put a box around a sentence. For example:

$\boxed{\text{If B, then } \bigcirc}$

"There are several other pieces in our set of attribute blocks. What piece can I add to the collection you see so that ONLY THIS STATEMENT will no longer be true?" (any non circle blue piece will work). Add one of these to the collection.

"Which part of the statement, if B or then ○, did we make false to make the whole sentence false?" Remove the piece last added.

Put a different statement in a box.

"What piece in the set can I add to the collection to make only THIS statement FALSE?"

Repeat this activity until the students can (1) find the statements TRUE of any small collection of the 24 piece set, and (2) find a piece to make FALSE any chosen TRUE statement.

Give pairs of students worksheets to complete, using attribute blocks if needed.

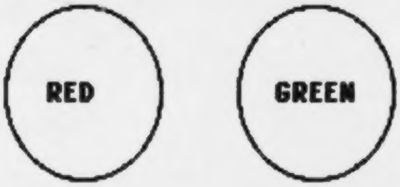
Give pairs of students worksheets to complete, using attribute blocks if needed.

LESSON FOUR: Use of OR (Exclusive)

Background: There are two kinds of OR. The most common is "exclusive or", or either-or. Satisfying one of the conditions linked by OR precludes satisfying the other. Example:

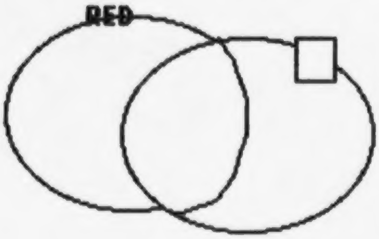
> He lives in Ely or Center City. If a person lives in Ely, he can't live in Center City. If living in Center City, he can't live in Ely.

This kind of OR is diagrammed as follows:



If an object is red, it can't be green and vice versa.

The other kind of OR is AND/OR or "inclusive or". This includes the possibility of satisfying BOTH conditions linked by the OR. This is diagrammed as follows:

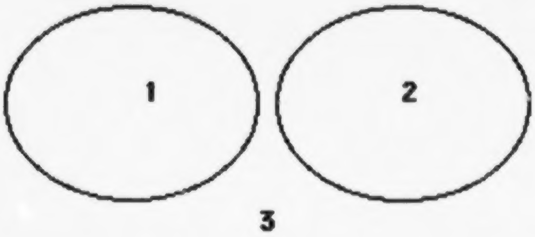


There are RED $\boxed{s}$, or things that are RED AND square.

One can usually distinguish between these two by looking at the conditions. If both involve the SAME property, it is EXCLUSIVE OR. An object can't be red and green at the same time. If the conditions involve DIFFERENT properties, it is INCLUSIVE OR. An object can be red and square at the same time.

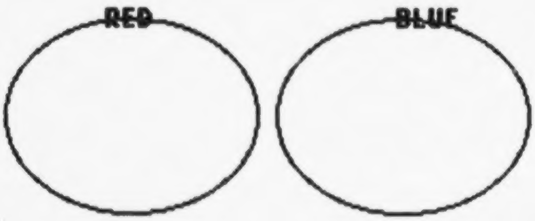
Students will not immediately see the difference between these, so this lesson should be repeated and the use of "or" analyzed where it appears in print regularly.

Introduction: Place two loops on the overhead projector:



"If an object is in 1, can it be in 2 or 3 at the same time?"

"It is either in 1 or 2 or 3." Change these labels:



"If we sorted the attribute blocks, they would go into 3 distinct groups. No piece would belong to 2 groups at the same time."

"We say a piece is red or blue or green and mean if one color, it can't be one of the other colors."

"This is called EXCLUSIVE OR".

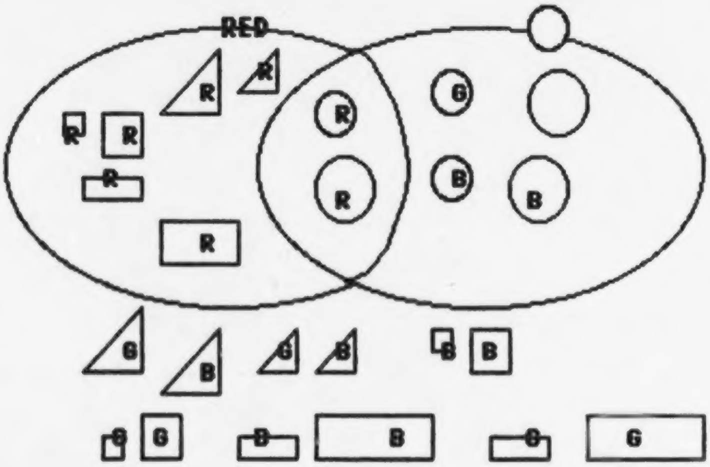
"For each sentence, I want you to tell me if the OR used is EXCLUSIVE OR."

> "Red or blue
> blue or green
> blue or square
> green or triangle
> rectangle or blue
> circle or triangle
> circle or square
> square or green
> triangle or red"

Assign the worksheet for students to work on in pairs.

LESSON FIVE: Use of OR (Inclusive or)

Introduction: Arrange the 24 pieces as shown:



"How many pieces are within BOTH of the loops?" (12)

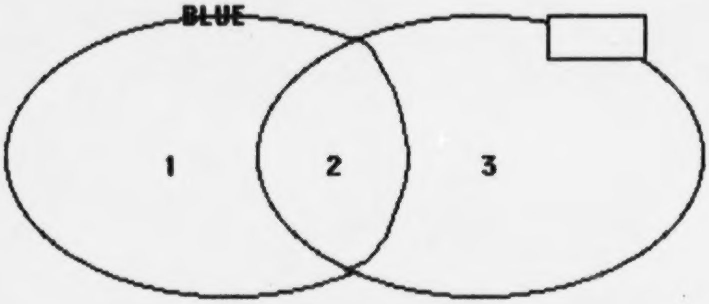
"Are the red circles included in this collection?"

"Do red circles belong to the RED group and the CIRCLE group AT THE SAME TIME?"

"Can everything inside the loops be described as RED OR CIRCLE?"

Note that those inside the loops that cannot be described by RED can be described by CIRCLE so ALL are described by RED OR CIRCLE.

Put this on the overhead:



"Which pieces go inside of the loops?" Place all pieces inside the regions 1, 2, or 3 as suggested.

"These pieces are all BLUE OR RECTANGLE."

Point out that this use of OR includes those that are AND, so it is sometimes used as and/or.

Give some examples, such as "boys or baseball players."

"Is it possible to be a boy and a baseball player AT THE SAME TIME?"

"This is INCLUSIVE OR."

Ask the students for phrases including or that are inclusive or because something can satisfy both descriptions at the same time.

Assign worksheets for pairs of students to complete.

LESSON SIX: Some Fun With Logic

Here are some activities to use at different times to involve students with logical thinking.

Tom has two glasses of the same kind. One is half full of water, the other is half full of Kool Aid. Tom takes an ounce of Kool Aid and mixes it into the water glass. Then he takes an ounce of the mixture and mixes it into the Kool Aid glass. Is there more Kool Aid in the water glass or water in the Kool Aid glass? Why?

Two boys look exactly alike, dress exactly alike, have the same date of birth, live at the same address, have the same parents and yet are not twins. How can this be?

Joanne needs five liters of water. She has a three-liter bottle and a four-liter bottle. She has no other containers to use. How can she obtain five liters in these bottles?

Bob worked at a grocery store. In the storeroom he found 3 boxes of nuts with three labels, "walnuts", "peanuts", and "walnuts and peanuts." He stuck the labels on the boxes without looking inside. The manager opened a box and told Bob ALL of the boxes were marked incorrectly,. How did he know? How could he replace the labels without opening all of the boxes?

At the school race, six girls ran in the 200 meter run. As the winner broke the tape, the following was noted:

1. Florence was 25 meters behind Jane.
2. Jane was 15 meters ahead of Paula
3. Barbie was in a tie with Jackie
4. Paula was 30 meters behind Francie.

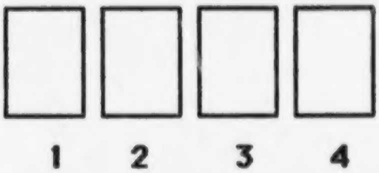
Who was 5 meters ahead of Jackie? List the order the girls were in as the winner crossed the finish line.

Barbara, Joan, Lil and Grace are in the 5th grade at the Washington school. Each has two outstanding qualities, but no two have the same two qualities. Two are very athletic; two are very pretty; two are very bright; and, two are very studious.

From this information find the two outstanding qualities of each girl.

1. Grace is very bright, but Barbara is not
2. Lil is very athletic, but Joan is not
3. Joan is very studious, but Barbara is not
4. Barbara is very pretty, but Lil is not
5. Lil has no quality that Joan has.

Four playing cards are in order, but upside down:



1. An ace is just to the right of a jack
2. A jack is just to the right of an ace
3. An ace is just to the right of an ace
4. A club is just to the right of a club
5. A club is just to the right of a diamond
6. A diamond is just to the right of a diamond

Name the cards that are in positions 1, 2, 3 and 4.

Write NEW DOOR on the board. Tell the children to make one word from these letters.

During lunch, Gladys, Terry, and Fred ate lunch together. One had a hamburger; one had a hot dog; and, one had a pizza burger.

1. Fred did not have the pizza burger or hot dog.
2. Terry did not eat the pizza burger.

Who ate what for lunch?

Jean, Dora, Irving, Shawn and Tony were put on a decorating committee. They sat around a round table in the above order and decided to elect a chairperson.

The first ballot was indecisive since each member got a single vote. Each person said, "I didn't vote for myself or either of my neighbors."

They voted again. Each voted the same as before except that Irving voted for Tony. This made Tony the chair. Who voted for Dora on the first ballot?

LEVEL FIVE

PROBLEMS INVOLVING MONEY

Background: In working with money, students should learn to use estimation and rounding techniques to obtain mental answers. Some examples follow:

In adding or subtracting:

Consider 42 adding and subtracting 1
+59
gives 41 a much easier mental
+60 computation.

Another is 42 adding 3 to both
-27 gives
45 a much easier mental
-30 computation

In multiplying or dividing using 25¢ or 50¢, students should use the facts 25 = 100/4 and 50 = 100/2 since multiplication by 100 is so easy mentally

$.25 x 482 then becomes 482 ÷ 4 = $120.40
$.50 x 261 becomes 261 ÷ 2 = 130.50

Multiplying and dividing factors gives easier computations that can be done mentally.

40 x 36 = 20 x 72 = 10 z 144 = 1440
65 ÷ 5 = 130 ÷ 10 = 13
118 ÷ 25 = 11800 ÷ 4 = 2950

In working with money, so many prices end in xxx9 that rounding and subtracting works. Consider:

.39 x 7 = .40 x 7 - 7 =$2.73
or 1.99 x 6 = 2.00 x 6 - 6 = 11.94
or .29 x 9 = 2.70 - 9 = 2.61

If adding several dollar amounts, round to dollars, add, then mentally subtract differences.

3.89 + 6.79 + 9.99
4 + 7 + 10 - .11 - .21 - .01
21.00 - .33 = $20.67

LEVEL FIVE

EXPONENTS AND SCIENTIFIC NOTATION

Background: Prior to using exponential notation, which arises in the interpretation of calculator displays, students must understand the use of exponents, especially as related to decimals and scientific notation.

LESSON ONE: Exponents

Introduction: Write: $5 \times 5 \times 5$ on the overhead or chalkboard. "How many times is 5 used as a multiplier?" Write: $5 \times 5 \times 5 = 5^3$

"The raised 3 shows how many times 5 is used as a multiplier."

Write: $10 \times 10 \times 10 \times 10$ on the overhead or chalkboard.

"What number should I write with the 10 to show it is used as a multiplier FOUR times?" Write: $10^4 = 10 \times 10 \times 10 \times 10 = 1000$

"How many 0's follow the 1 in the numeral? How do we write this as a POWER OF TEN?"

Write: $100 = 10?$ (10^2)
Then: $1000 = 10^?(10^3)$
Then: $100000 = 10^?$ (10^5)

Write: 2x2x2 X 2x2

"How many times is 2 a multiplier in the first number? in the second number? in the number that is the product?" (3, 2, 5)

"What is this result?" $8 \times 4 = 32 = 2^5$
Consider: $10 \times 100 = 1000$

"Each number written with an exponent is $10^1 \times 10^2 = 10^3$

"What do you do with the exponents when multiplying numbers made up of the same multipliers?"

$A = 10^2$

$B = 10^4$

"What is the result of multiplying A by B?" (10^6)

Pass out the worksheets for pairs of students to work on.

LESSON TWO: Separating Numbers into Powers of Ten

Introduction: Write: 600 on the overhead or chalkboard.

"Let's see how many ways we can write this as a product of two numbers."

$600 = 2 \times 300$
$600 = 3 \times 200$
$600 = 4 \times 150$
$600 = 5 \times 120$
$600 = 6 \times 100$

"Notice we have a small number less than 10 multiplied times a power of 10." Write: $600 = 6 \times 10^2$

Write: 625 on the board or overhead. "How do we divide this by 10?"

$$\begin{array}{r|l} & \ \ 62\ 5/10 \\ \hline 10 & 625 \\ & \underline{60} \\ & \ \ 25 \\ & \ \ \underline{20} \\ & \ \ \ \ 5 \end{array}$$

"Written as a decimal this is 62.5

Write $625 = 62.5 \times 10$

"The most useful form is to have a small number less than 10 times a power of ten. How can we divide 62.5 by 10?"

$$\begin{array}{r} 6.25 \\ 10\overline{)62.5} \\ \underline{60} \\ 25 \\ \underline{20} \\ 5 \end{array}$$

"So 62.5 = 6.25 x 10"

"Now we can write 625 is 6.25 x 10 x 10 or 6.25 x 10^2."

"Remember that dividing by 10 changes each place to the next smaller, so 625 ÷ 10 = 62.5

6 in the hundreds place goes to the tens place
2 in the tens place goes to the ones place
5 in the ones place goes to the TENTH place."

"Dividing by 100 changes each place to the second smaller, so
625 ÷ 100 = 6.25 x 10^2

6 goes hundreds to ones
2 goes tens to tenths
6 goes ones to hundredths."

"How would we write 387 as a number between and 10 times a power of ten?"

We get the number between one (one digit) by dividing by 100 so 387 = 3.87 x 10^2."

"Notice that multiplying on the right side gets back to the 387."

Write: 1.23 x 10^3

"How do we write this as a single numberal? One way is by 10 at a time:

$$\underset{(1)}{1.23 \times 10} = \underset{(2)}{12.3 \times 10} = \underset{(3)}{123 \times 10} = 1230$$

so 1.23 x 10^3= 1230

"To Summarize: To multiply by powers of ten, move the decimal point the exponent number of places to the RIGHT. To divide by powers of ten, move the decimal point the exponent number of places to the LEFT."

This is why having numbers in the form of a small number multiplied by powers of ten is so important to know."

Consider: $2 \times 10^3 = 2000$
$\underline{3 \times 10^5} \times \underline{300000}$
$6 \times 10^8 \quad 600000000$

Pass out worksheets for students to work on in pairs.

LEVEL FIVE

USING THE CALCULATOR

Background: Children should be familiar with the keyboard of a simple calculator with memory. At this level, they should use it to solve problems that involve decimals.

LESSON ONE: Introduction

"Tracy is trying to sell an item that has been in the store a long time. She decides to set the price each day as .9 of the price the day before. What is the price on the fifth day of the sale? for the item that regularly is $42.00?" "What do we enter first?"

KEY $\boxed{4}\boxed{2}$

"What operation do we use to find .9 of this price?"

KEY $\boxed{\times}$

"What number is the multiplier now?"

KEY $\boxed{.}\boxed{9}$

"What is the price during the first day of the sale?" Enter this in the table below:

Sale Day	1	2	3	4	5
Price	37.80				

"What do we do with this price to get the new price for the second day?"

Press $\boxed{\times}\boxed{.}\boxed{9}$ **and enter 34.02 into the table.**

At this point, you might want to show students the repeating multiplier feature. Some calculators hold the first factor entered as a multiplier, others the second factor. The sequence of keys must be set according to the calculator feature.

Example:

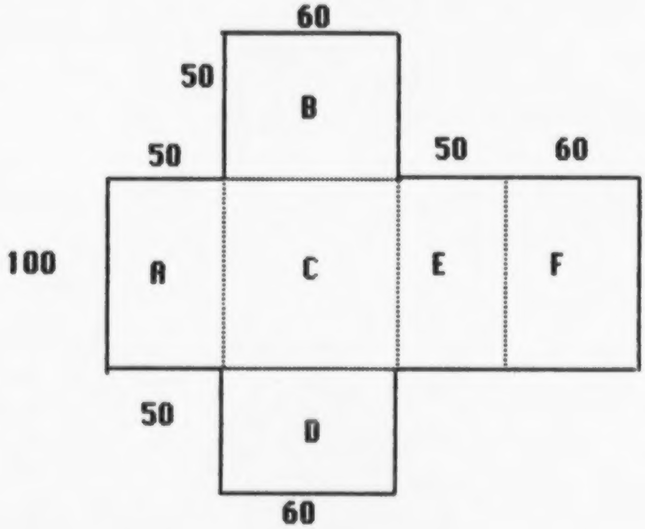
FIRST FACTOR HELD	SECOND FACTOR HELD
.9 x 42 =	42 x .9 =
= for each successive multiplication by .9	= for each successive multiplication by .9

Continue until the table is filled and the price during the fifth day is found as $24.80. Assign worksheets to be used with calculators by pairs of students.

LESSON TWO: Using Memory

Background: Using memory is helpful when a number of products or quotients are to be added or subtracted and in different combinations. As you recall, order of operations required renaming (finding products and quotients) before adding or subtracting the numbers.

Introduction: Put an overhead transparency of the following on the overhead projector:



These can be folded to make a box. To find how many square inches of cardboard are needed, we have to find the area of each rectangle, then add these together. The M+ key of the calculator helps to do this. Let's see how:

"How do we find the area of rectangle A?"

KEY SEQUENCE

$\boxed{5}\boxed{0}\boxed{\times}\boxed{1}\boxed{0}\boxed{0}\boxed{=}$

"Is there another rectangle like this one? (E) We double the area."
$\boxed{5}\boxed{0}\boxed{\times}\boxed{1}\boxed{0}\boxed{0}\boxed{=}\boxed{\times}\boxed{2}\boxed{=}$

"We put this product in memory while we find the others."
$\boxed{5}\boxed{0}\boxed{\times}\boxed{1}\boxed{0}\boxed{0}\boxed{=}\boxed{\times}\boxed{2}\boxed{=}\boxed{M+}$

"What is the area of rectangle B?"

"Is there another like this? (D)"

"KEY this in and put into memory to be added to the last product."

"What is the area of rectangle C?"

"Is there another with the same dimensions and area?" (F)

"KEY in this and put into memory+."

"Now all three products have been added in memory." We see what this sum is by: $\boxed{RCM}$ This will put what is in the memory on the display so we can see it."

Do a second "geojacket" to illustrate the use of $\boxed{M+}$ and then assign the worksheets provided for use with calculators.

SOME CALCULATOR EXPLORATIONS FOR STUDENTS

"How long does it take you to count to 1000 on the calculator? Keep a record of times for counting by 2, 3, 4, 5, 6, 10, 15, 25, 100."

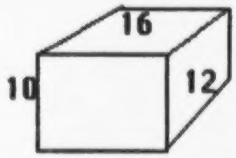
"For which numbers between 125 and 300 is the sum of the digits 16?"

"Round each number to the nearest 100 and find the sum:
12345
132465
142362
23456
324651
56234"

"Where would you open a book so the product of the page numbers of pages facing each other is 16512?"

"Which of these have the same answer:
26 x 29 131 + 286 + 245 + 103
957 - 203 3020 ÷ 4"

"Design a box with different dimensions with the same volume as this one:



"Find these batting averages.

Player	Hits	Times at Bat	Batting Average
Puckett	211	600	
Gaetti	197	603	
Lombardozzi	183	589	
Hrbek	164	590	

"Use the calculator to find the missing digits:

$$\begin{array}{r} 46 \\ 7\square \\ \square 8 \\ \hline 160 \end{array} \qquad \begin{array}{r} 9\square \\ -\ \square 4 \\ \hline 56 \end{array}$$

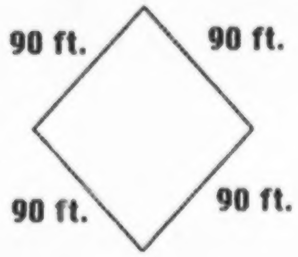
$$32\square \times 6\square 1 = 194724$$

$$187\square \div \square 5 = 41.666666$$

"Suppose you could spend $2.00 every minute, day and night, for a year. How much would you spend?"

"Jim's car moves forward 5.8 feet each time the wheels go all the way around. If his tires are guaranteed for 60,000 miles, how many times should the wheels turn around during the guarantee period?"

"The dimensions of a baseball diamond are shown:



Last year Kent Hrbek hit 47 home runs. How many miles did he run going around the bases?"

"Which is the better buy for cereal?

14 oz. for $1.39
20 oz. for $1.99
28 oz. for $2.59?"

"Complete this table:

+	36	359		726
106	142			
				1578
		875		
993				

"Do the division indicated by these fractions. Show the decimal to as many places as on the calculator."

FRACTION DECIMAL
1/9 =

2/9 =

1/3 =

4/9 =

5/9 =

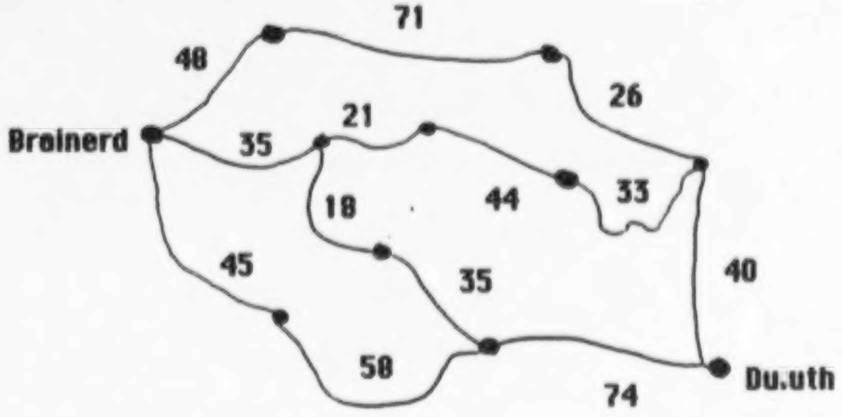
2/3 =

7/9 =

8/9 =

FRACTION	DECIMAL
1/4 =	
1/2 =	
3/4 =	
1/5 =	
2/5 =	
4/5 =	
1/6 =	
5/6 =	
1/8 =	
3/8 =	
5/8 =	
7/8 =	
1/12 =	
5/12 =	
7/12 =	
11/12 =	

"What is the shortest way from Brainerd to Duluth?"



"The mileage on the odometer on my car is shown:

0	9	4	2	7	3

"How many miles must I travel before all digits are the same?":

"How many miles must I travel before the number of miles is divisible by 100?"

"How many miles must I travel before the digits will be shown in reverse order?"

"The year in which you were born is divisible by what numbers?"

"Which two numbers adding to 120 have the largest product?"

"Frank is a 'wiseguy.' When someone asked him how old he was, he answered, '5,785,560 minutes.' How many years old is Frank?"

"Jim buys and sells baseball cards. He bought a Hank Aaron for $5, sold it for $8, bought another Hank Aaron for $16.50, sold it for $28.75. How much did he make buying and selling Hank Aaron cards?"

"On a ticket roll, ticket numbers 10, 11 and 12 add to 33. What 3 consecutive ticket numbers add to 372?"

"Tom bought 10 cans of peaches at 49¢ per can and 2 boxes of cereal. He received $2.80 in change. What was the cost of each box of cereal?"

"How many hours of your life have you spent watching TV?"

USING LOGO

See the Level Four lessons for the kinds of commands the students will have had prior exposure to. The primary new idea to be introduced at this level is one of the most powerful in LOGO or any other structured language - RECURSION. Recursion takes place when a procedure calls itself, rather than another procedure, as in: REPEAT 4 [SIDE RT 90]

WHEN a procedure calls itself, it must know when to stop doing this. The most common way of doing this is to use an IF test that transfers control out. Another is to set a limit on the number of times the call is made. This is done by creating a variable that acts like a counter.

Example 1: Using an IF test and a counter. Required is a subprocedure BOX

```
TO BOX : SIZE
REPEAT 4 [FD : SIZE RT 90]
END

TO SPINBOX : COUNTER : SIZE
IF : COUNTER = 0 [STOP]
BOX : SIZE
RT 45
SPIN BOX : COUNTER - 1: SIZE
END
```

: COUNTER is a variable that must be assigned a value when the procedure is called. It is the number of time : the procedure is to repeat itself.

: SIZE must also be assigned a value. It specifies the length of the side of the box made.

SPINBOX 5 40
    The box will turn RT 45   5 times. The box side will be 40.
SPINBOX 6 50
    The box will turn RT 45 6 times. The box side will be 50.

Each time the box is is made the counter goes down 1. When it reaches 0, the procedures STOPS. This can be diagrammed as follows:

```
SPINBOX 4 50
  IF 4 = 0 [STOP]
  BOX 50
  RT 45
   SPINBOX 3 50
     IF 3 = 0 [STOP]
     BOX 50
     RT 45
      SPINBOX 2 50
        IF 2 = 0 [STOP]
        BOX 50
        RT 45
         SPINBOX 1 50
           IF 1 = 0 [STOP]
           BOX 50
           RT 45
            SPINBOX 0 50
              IF 0 = 0 [STOP]
           END
        END
     END
  END
```

Once students have a knowledge of what the primitive commands do, it becomes a matter of trying these out to see what happens. You can introduce them to other primitives associated with turtle position - HEADING, POSITION, HOME, etc. to help them make more efficient programs. Some good classroom resources for you include:

The Turtle Sourcebook
  Bearden, Martin & Muller
    Reston Publishing Co.

LOGOWORLDS
  Babbie

Wadsworth Publishing Co.

Apple LOGO Programming and Problem Solving
Billstein, Libeskind & Lott
Benjamin Cummings Pub. Co.

Learning Math With LOGO
Neufeld
LOGO Publications, London; Ontario

Exploring with LOGO, A Series
Campbell, Fenwick
Allyn and Bacon, Inc.

LEVEL FIVE

USING DATA

LESSON ONE

"Several students in Mrs. Hanson's class volunteered to count the cars that went past the school. They each watched for 15 minutes and recorded the results. The table shows what 30 children counted."

Students	Cars Counted
Tom	4
June	9
Willie	6
Frank	8
Ruth	11
Joanne	13
Bob	7
Jackie	5
Gloria	10
Tammy	9
Sammy	8
George	4
Joyce	11
Barbara	3
Grace	5

"Who counted the most cars?"

"Who counted the fewest?"

"What was the difference between the fewest counted and the most counted?"

"This is called the RANGE of the data."

"What was the number of cars counted most frequently?"

"This is called the MODE of the data."

"What do you think the average number of cars counted was?"

"Let's put the data into an arrangement where we can see it better. Remember how we arranged data in 'Guess My Rule'?"

"What was the greatest number of cars counted?"

Write: Joanne 13

"What was the next largest number?"

As students continue supplying these in order, write them:

Joanne	13
Ruth	11
Joyce	11
Gloria	10
Jane	9
Tammy	9
Frank	8
Sammy	8
Bob	7
Willie	6
Jackie	5
Grace	5
George	4
Tom	4
Barbara	3

"Now what do you think the average is?"

"What is the middle number?"

"The ARITHMETIC AVERAGE is the sum of the numbers divided by the number of numbers."

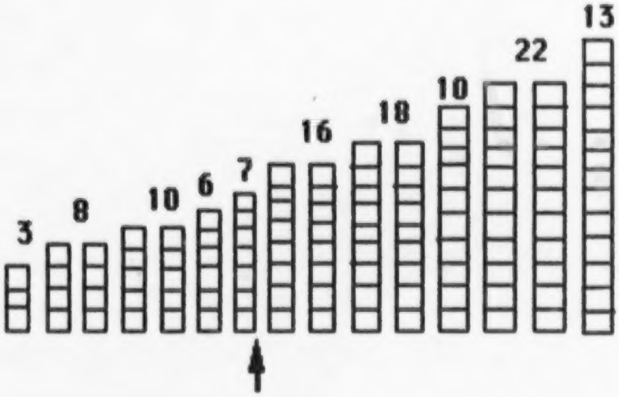
"Use your calculator to find the average of these 15 numbers." (7.533)

"Is this the number of cars anyone counted?"

"Could you count this number of cars?"

"Does this number or the middle number give you a better idea of the 'average' number of cars in a 15 minute period?"

Graph the data using UNIFIX cubes.



"The arrow points to the average you computed."

"Do you think there are more Unifix cubes to the left or right of this place?"

"Now look at HOW MANY numbers are to the right and left?"

"Obviously AVERAGE is an artificial number of some kind. It is the definition of the sum of numbers ÷ the number of numbers."

"The average of 1 & 3 is 2; 4 ÷ 2

The average of 1, 3 and 5 is 3; 9÷3."

"Notice it is two away from 3 as is 5."

"The average of 1, 3, 5, 7 and 9 is 5."

"Notice 3 & 7 are just as far from 5 and 1 and 9 are just as far from 5."

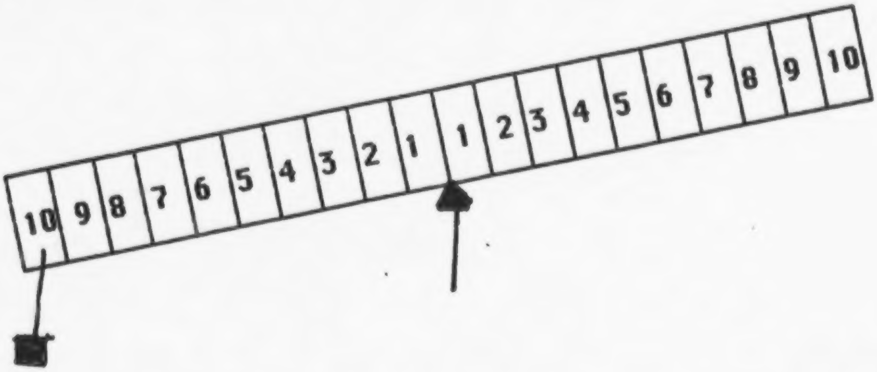
"Perhaps the distance that numbers are from the average is important."

"We'll see more about that later."

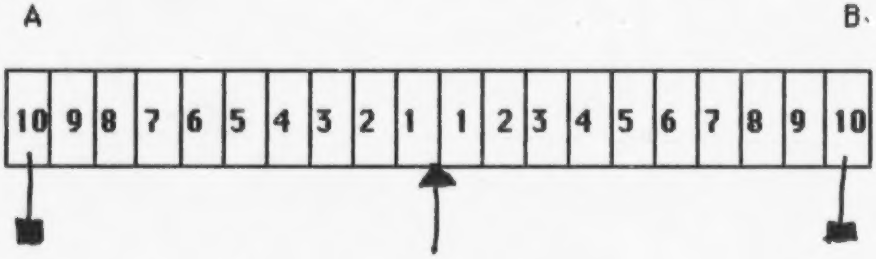
Activity: Assign pairs of students with calculators to complete the recording forms.

LESSON TWO

Introduction: Use beam balance that has places to hang weights or to insert weights.

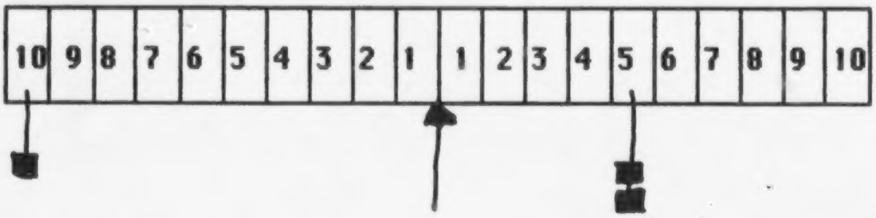

10
9
8
7
6
5
4
3
2
1
1
2
3
4
5
6
7
8
9
10

Place a weight as shown. "Where do I put a weight on the other side so this will balance?"


A
B
10
9
8
7
6
5
4
3
2
1
1
2
3
4
5
6
7
8
9
10

Take the weight from B. "I want to put TWO weights on that side. Where do I put them?"

Place as children suggest. If not at 5, follow up by asking why they think it doesn't balance? Eventually get to:


10
9
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1
1
2
3
4
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6
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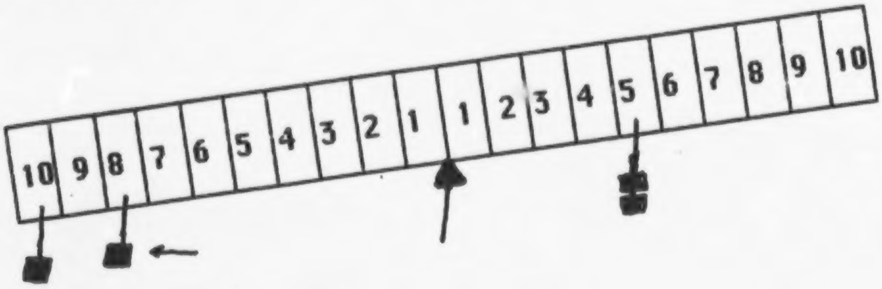
8
9
10

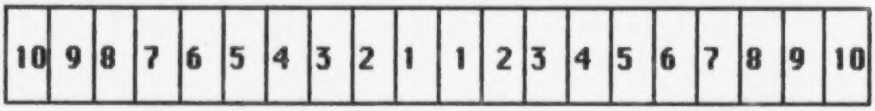
"How can TWO weights on one side balance ONE weight on the other?"

Discuss importance of DISTANCE the weight is from the center. Place a weight as shown:



"Why does the beam go down when there are TWO weights on each side?" Discuss.

"Where should I put TWO weights on the other side so the beam will balance again?" The eventual outcome of this interaction should be:



"Notice that we have on one side 10 x 1 + 8 x 1 = 18. On the other side we have 4 x 2 + 5 x 2 = 8 + 10 = 18."

"The turning caused by the weights depends on the distance they are from the center."

Activity: Have students work in pairs on the worksheet.

LESSON THREE

Introduction: "Here is a set of test scores." Write the following on the chalk board or overhead:

Ronald	75
Sally	80
Bill	60
George	65
Grace	75

"What should we do with these first?" Put them in order:

Sally	80
Grace	75
Ronald	75
George	65
Bill	60

"What is the average of these scores?" (80 + 2(75) + 65 + 60 ÷ 5 = 71

"Consider a beam with 71 as center and the scores as weights on the beam. What is the range of scores?" (20 points)

"We'll make the beam 21 units long ranging from 60 to 80. It is 21 units so 60 and 80 can be included."

60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80

"Put the center at the average - 71."

60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80

"Each score is a weight."
"How far is 65 from 71?" (6)
"How far is 60 from 71?" (11)
"How far is 75 from 71?" (4)
"There are how many weights at 75?" (2)

"How far is 80 from 71?" (9)

"Now we check the weights x distance from center on each side."

$$(6 \times 1 + 11 \times 1 = 17 \quad (4 \times 2) + 9 = 8 + 9 = 17$$

"The turning effect of the weights is the same on one side as on the other side so it is a balanced beam."

"The average you find of the scores is the center of a beam of the length of the range of scores that is in balance."

Here's another set of numbers:

Alice	36 points
Joyce	18 points
Agnes	17 points
Terri	13 points
Janine	11 points

"What is the middle number?" (17)

"What is the average number of points?" (19)

"What is the range of points?" (25)

"We'll make a beam 25 units long."

11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36

"Where does the center go?" (19)

"Where are the weights placed?"

11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36

"Notice that only one number is above the average, while 4 are below it."

"Why do you think that is so?"

"Why is the average away from the middle number?"

Point out the turning effect of the 36 is 17 x 1 since there is just one 36. The turning effect in the opposite direction is:

1 x1 + 2 x 1 + 6 x 1 + 8 x 1 = 1 + 2 + 6 + 8 = 17, so the beam balances.

"You have to be careful using the average because very big values or very small values affect it. Sometimes the middle number better describes the data.

Activity: Have pairs of students complete the worksheet.

TAKING A CHANCE

Background: Students should begin to get a feel for probability and the likelihood of certain outcomes. The Probability of an event (E) is estimated by $P(E) = \frac{\text{The number of observations favorable for (E)}}{\text{The total number of observations}}$.

For example, a die has 6 faces numbered 1, 2, 3, 4, 5, 6. If it is not loaded, each of these 6 numbers has an equal chance of showing on the top face when it is rolled. There are 6 possible numbers to show so the probability of rolling a 6 is $1/6 = \frac{\text{outcomes favorable}}{\text{total outcomes}}$

In rolling 2 dice, getting a 12 requires a 6 on each die. That is 6 AND 6 on the dice. The probability of 6 on one is 1/6; on the other, also 1/6, so the probability of getting 12 as 1/6 x 1/6 = 1/36. Once in 36 times, ON THE AVERAGE, or over MANY trials, a 12 will show. This does NOT mean that if NOT 12 shows on 35 rolls a 12 MUST show on the 36th!!!.

LESSON ONE: Coins

Introduction: There are 2 ways a coin can turn up when flipped - HEADS or TAILS. Have each student bring a coin of any denomination to class. Give each student a record form. One is provided in the black line masters.

	Tally	**Total**
Heads		
Tails		
Total		**50**

Each student is to flip a coin 50 times tallying the number of heads and tails. Each student is also to calculate the fraction of the 50 tosses that were heads and the fraction that were tails. Make a chart on which students can record the fractions obtained. Discuss the different fractions.

"All of these fractions are close to what simple fraction?"

Have the students use calculators to TOTAL the number of HEADS for ALL students. Do likewise for the TAILS. 50 x number of students in the class is the TOTAL TRIALS.

Now compute the fractions: $\frac{\text{TOTAL HEADS}}{50 \times \text{no. of students}}$

$\frac{\text{TOTAL TAILS}}{50 \times \text{no. of students}}$

These fractions should be even closer to 1/2.

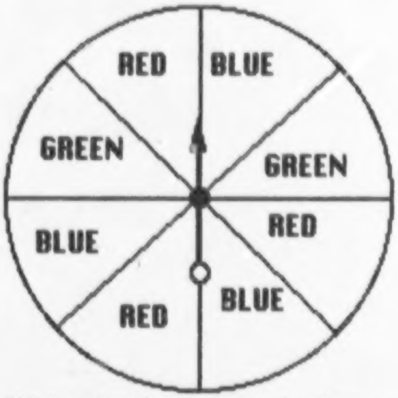
"A probability estimate is based on an experiment like this. It can change from one experiment to the next. The more trials in the experiment, the more accurate it will be. A probability Estimate gives you a reasonable expectation of what might happen."

"In the case of fair coins, there are just 2 possible outcomes of a flip - HEADS or TAILS. Each has an equal chance of coming face up, so the probability of a HEADS is 1/2 and of a TAILS is 1/2.

Activity: Give the worksheet to pairs of students to work on.

LESSON TWO:

Place a spinner transparency like that shown on the overhead. Use colored transparency film to make it to help the students see how much of it is each color:



"If I spin the arrow, is it more likely to point to red or to green?"

"Why do you think so?"

"Discuss 3/8 of the circle vs. 2/8 of the circle. "

"Is to more likely to point to blue or to green?"

:Is it more likely to point to blue or to red?: (equally likely)

"If you spin the spinner 40 times, how many times would you expect it to point to red? (15) to blue? (15) to green? (10)"

"What is the probability estimate of landing on red? (3/8) on blue? (3/8) on green? (2/8 or 1/4)"

"If you spin it and it lands on red, what is the probability estimate of landing on red on the next spin?"

Discuss this thoroughly. The probability estimate is NOT affected by past outcomes. If you flip H, H, H, H, , the probability T on the next flip is STILL 1/2! Some students think there is such a thing as "the law of averages catching up." These are independent events and the outcome on one has no effect on the outcome of the next trial.

Have pairs of students use spinners to record the results of 100 spins. Make a chart of all these records like you did with coin flips and work the data in the same way as you did there. The class fractions for red and blue should be close to 3/8 or .375. The fraction for green, close to .25.

Activity: Have pairs of students use the worksheet to estimate probabilities for the situations given.

Verbal Multiplication and Division Problems: Some Difficulties and Some Solutions

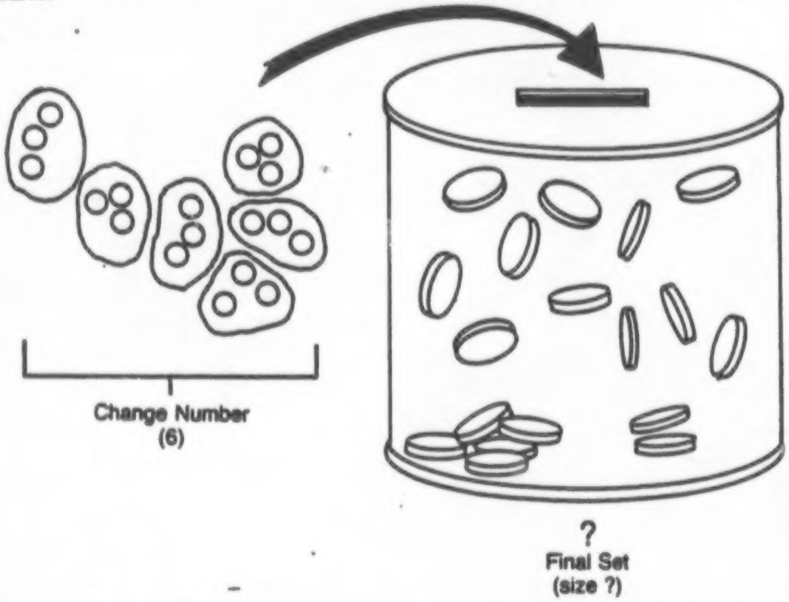
By **A. Dean Hendrickson**

Verbal problems that involve multiplication and division are difficult for children to solve. Many of these difficulties arise because of their limited understanding of these arithmetic operations. Their experience with the different kinds of situations that call for these operations is also limited. At the same time, these problems cannot be categorized easily because the situations that require these operations are varied. Nonetheless, multiplication is often taught only as "repeated addition" and division only as "repeated subtraction." Children must have specific instruction in *all* the situations that require multiplication and division as arithmetic operations if they are to apply them successfully to verbal problems.

Change Problems

Extensions of the "change problems" for addition and subtraction can lead to multiplication and division. In this particular kind of problem we have an initial set, a change number, and a final set. Given an initial set of small size and a change number that describes how many of this size set are joined, we find the size of the larger final set by multiplication. These problems are *change 1*, or repeated addition, problems. Here is an example (fig. 1):

Fig. 1 An example of a *change 1* or "repeated addition" problem: John put 6 handfuls of 3 tokens each into a bank. How many tokens did John put in the bank?



> John put 6 handfuls of 3 tokens each into a bank. How many tokens did John put in the bank?

Change 2 problems result when a large initial set is given along with the size of a smaller final set, and a change number needs to be found that describes *how many* sets of that size can be made from the initial set. This problem represents the *measurement*, or repeated-subtraction, interpretation of division. Here is an example (fig. 2):

> Susie has 24 cookies. She gives 3 cookies to each of the children on the playground. How many children are on the playground?

A child who can reverse the "putting together" transformation can relate a measurement interpretation of the division of countable materials to the repeated-addition kind of multiplication. In some ways the division is easier, since the child must retain only the final set size and count the number of sets that can be made. The count is constructed in the process and the size of the initial set is not important, since the count stops whenever the process runs out of objects. In repeated addition, both the count num-

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ber and the size of the initial set must be retained mentally along with the result at the end of each successive joining.

Change 3 problems involve a large initial set and a known change number; the size of the final, *equal* sets that can be made from the initial set must be found. This is the *partition* interpretation of division. An example follows (fig. 3):

> Susie has 24 cookies. She gives an equal number to each of her 4 friends. How many cookies does each friend get?

Change 2, or measurement division, is easier, since only the size of the set being formed repeatedly must be retained and a count of these sets kept as they are made. Change 3, or partition division, requires a strategy to assure the equality of the sets being made and hence is more difficult.

Comparison Problems

Questions involving "less than" or "more than" lead to addition and subtraction problems. These problems involve a comparison set, a difference set, and a referent set. When we compare two sets and the comparison involves questions of "how many times as many" or "what part of," we use multiplication and division. Such problems involve a comparison set, a referent set, and a correspondence other than a one-to-one correspondence between these sets. In figure 4, if the question is asked, "*A* has how many times as many as *B*?" then *A* is the comparison set, *B* is the referent set, and the correspondence of *A* to *B* is sought.

Compare 1 problems result when the referent set and a many-to-one correspondence are given and students are asked to find the comparison set. The following is an example (fig. 5):

> Iris has 3 times as many nickels as dimes. She has 4 dimes. How many nickels does she have?

Multiplication is used to find the answer: 3 × 4 = 12.

Compare 2 problems occur when the comparison and a many-to-one

Fig. 2 An example of a *change 2* problem, measurement or repeated-subtraction interpretation of division: Susie has 24 cookies. She gives 3 cookies to each of the children on the playground. How many children are on the playground?

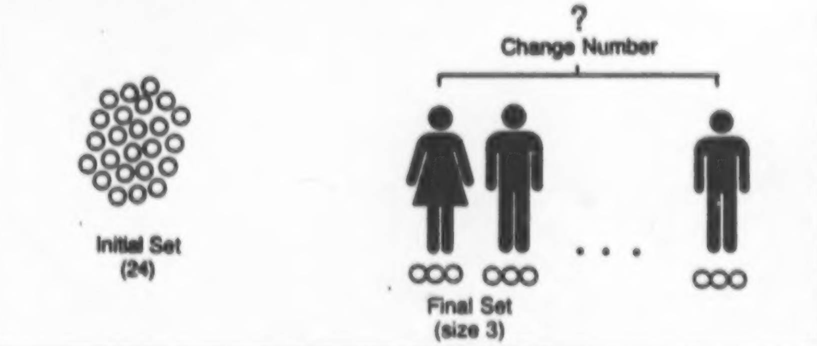


Fig. 3 An example of a *change 3* problem, a partition interpretation of division: Susie has 24 cookies. She gives them in equal numbers to her four friends. How many cookies does each friend get?

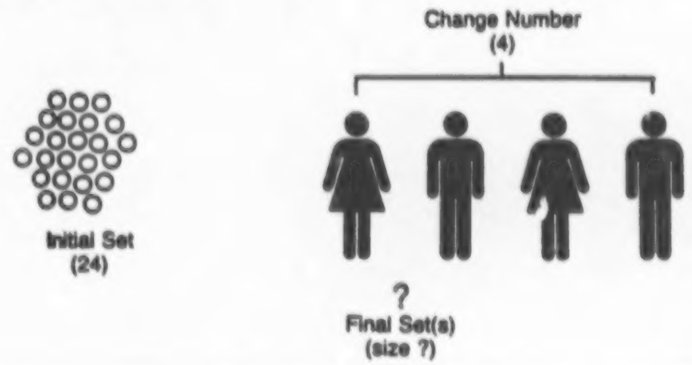


Fig. 4 A comparison problem: Find the correspondence of *A* to *B*. *A* has how many times as many as *B*?

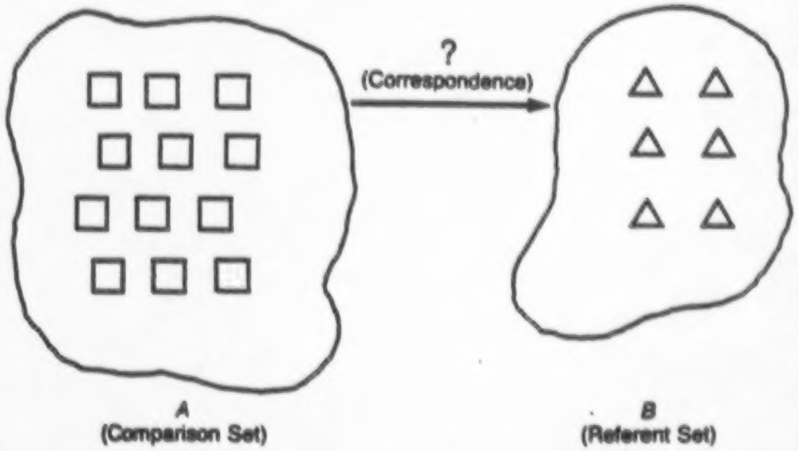


Fig. 5 A *compare 1* problem: Iris has 3 times as many nickels as dimes. She has 4 dimes. How many nickels does she have?

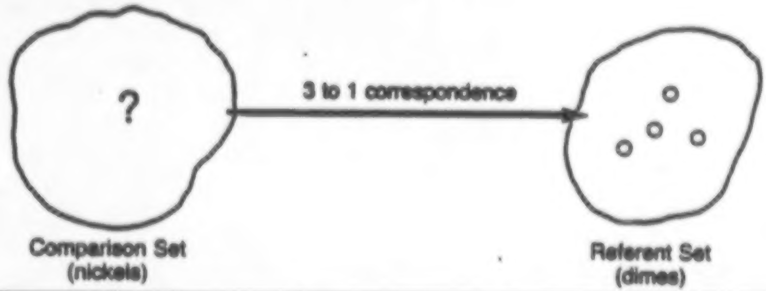

?
3 to 1 correspondence
Comparison Set
(nickels)
Referent Set
(dimes)

Fig. 6 A *compare 2* problem: Iris has 15 nickels. She has 3 times as many nickels as dimes. How many dimes does Iris have?

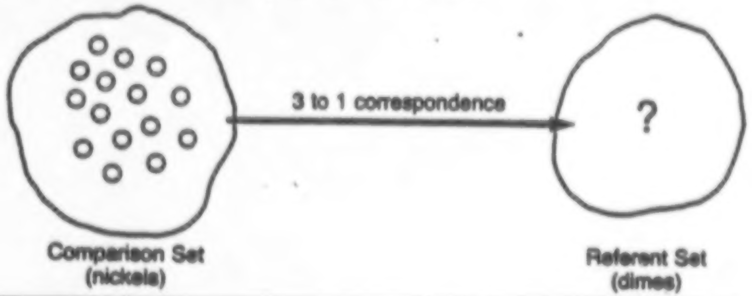

3 to 1 correspondence
?
Comparison Set
(nickels)
Referent Set
(dimes)

Fig. 7 A *compare 3* problem: Frank has 24 nickels and 8 dimes. He has how many times as many nickels as dimes?

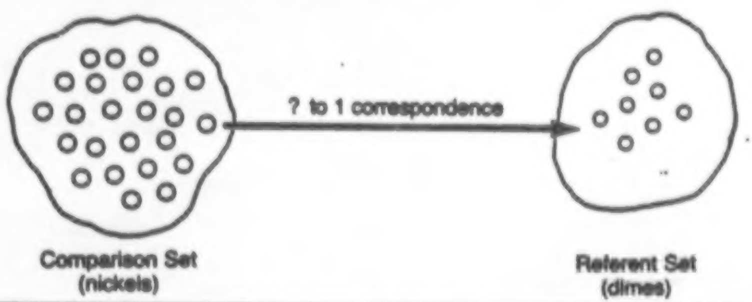
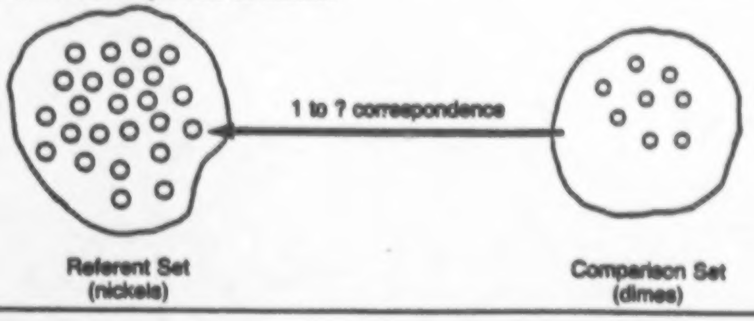

? to 1 correspondence
Comparison Set
(nickels)
Referent Set
(dimes)

Fig. 8 A *compare 4* problem: Frank has 24 nickels and 8 dimes. He has what fraction as many dimes as nickels?


1 to ? correspondence
Referent Set
(nickels)
Comparison Set
(dimes)

correspondence are given and the referent set must be found. Here is an example (fig. 6):

> Iris has 15 nickels. She has 3 times as many nickels as dimes. How many dimes does Iris have?

Division is used to find the answer: $15 \div 3 = 5$.

Compare 3 problems result when the comparison set and referent set are known and a many-to-one correspondence must be found (fig. 7):

> Frank has 24 nickels and 8 dimes. He has how many times as many nickels as dimes?

Division is used to find the answer: $24 \div 8 = 3$.

Compare 4 problems occur when a comparison set and a referent set are given and a one-to-many correspondence is sought. In this case, the comparison set is the smaller of the two. Here is an example (fig. 8):

> Frank has 24 nickels and 8 dimes. He has what fraction as many dimes as nickels? (or, Frank's dimes are what fractional part of his nickels?)

The result is division of a smaller by a larger number or formation of a rational number, usually expressed as a fraction: $8 \div 24 = 1/3$.

This kind of question puts a child's concept of *fraction* being equal parts of a whole into conflict with this ratio situation. What other language can be used to ask for this correspondence? Because of the difficulty of finding suitable language, questions related to finding this correspondence are seldom found in textbooks.

Compare 5 problems arise when the comparison set and the referent set are given and a many-to-many correspondence is sought (fig. 9):

> There are 12 girls and 16 boys in the room.. How many times as many boys are there as girls?

One divides to find the answer ($16 \div 12 = 4/3$). Here again a fraction tells how many times as much, although a ratio correspondence is made in the thinking.

Compare 6 problems occur when the comparison set is smaller than the referent set and the correspondence is sought (fig. 10):

> There are 12 girls and 16 boys in a room. The number of girls is what part of the number of boys?

The result is found by division again, 12 ÷ 16 = 3/4, and the same conflict between ratio and fraction results.

Compare 7 problems result when the larger comparison set and the many-to-many correspondence are given and the size of the smaller referent set is sought (fig. 11):

> There are 16 boys in a class. There are 4/3 as many boys as girls. How many girls are there?

The answer is found by dividing: 16 ÷ 4/3 = 12.

Compare 8 problems arise when the smaller referent set is given along with a many-to-many correspondence. The size of the larger comparison set is sought (fig. 12):

> There are 12 girls in the room. The number of boys is 4/3 the number of girls. How many boys are in the room?

The answer is found by multiplying 4/3 × 12 = 16.

The compare problems that involve many-to-many correspondences are difficult, since they bring into conflict the child's recognition of a fraction as comparing a given number of equal parts to the whole and the idea of ratio as a correspondence. The use of the same symbolism for both fractions and rational numbers compounds this difficulty.

Thinking in ratios, equating ratios, and applying ratios to situations involve formal operational thought. Very few elementary children are capable of this kind of reasoning. In fact, few eighth and ninth graders can think through the Mr. Tall–Mr. Short problem:

	Mr. Tall	Mr. Short
Measured in match sticks	9	6
Measured in paper clips	12	?

Fig. 9 A *compare 5* problem: There are 12 girls and 16 boys in the room. How many times as many boys are there as girls?

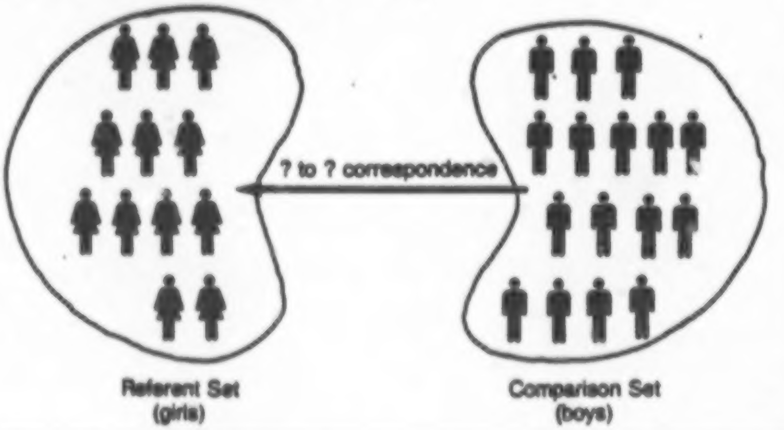


Fig. 10 A *compare 6* problem: There are 12 girls and 16 boys in a room. The girls are what part of the boys?

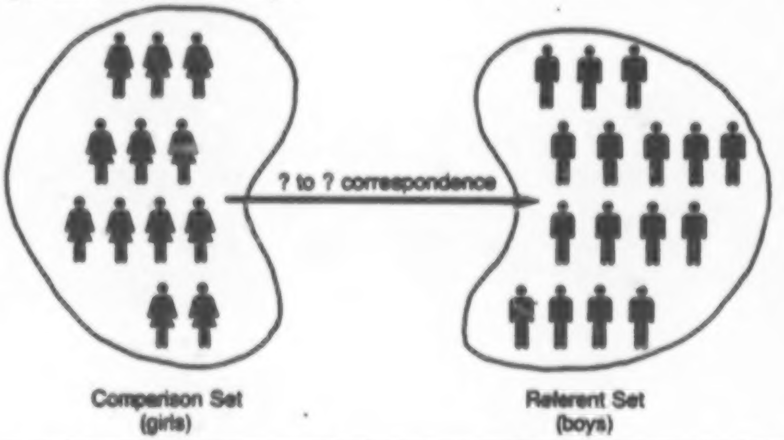


Fig. 11 A *compare 7* problem: There are 16 boys in a class. There are 4/3 as many boys as girls. How many girls are there?

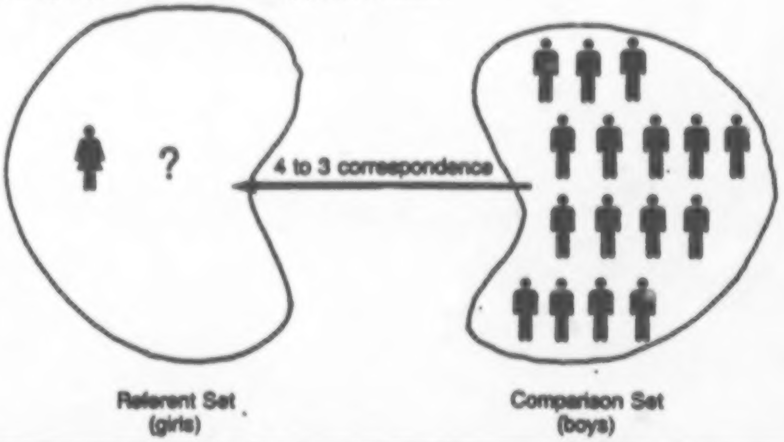
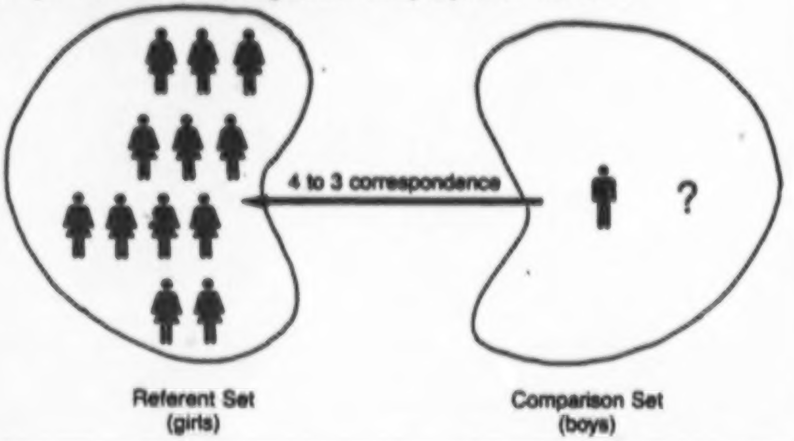


Fig. 12 A *compare 8* problem: There are 12 girls in the room. The number of boys is 4/3 the number of girls. How many boys are in the room?


4 to 3 correspondence
?
Referent Set
(girls)
Comparison Set
(boys)

Rate Problems

The kind of proportional reasoning used in equating ratios is also involved in thinking about rate problems. These are commonly found in intermediate textbooks. A rate problem involves two variables—one independent and one dependent—and a rate of comparison between them. An example is distance (miles) = rate (miles per hour) × time (hours). Here the number of hours is the independent variable, the distance in miles (a total) is the dependent variable, and the ratio of miles to hours is the rate.

Some common rate examples are these:

Fig. 13 A *rate 1* problem: Fred pays $12.00 a square yard for outdoor carpeting. How much will 16 square yards cost?

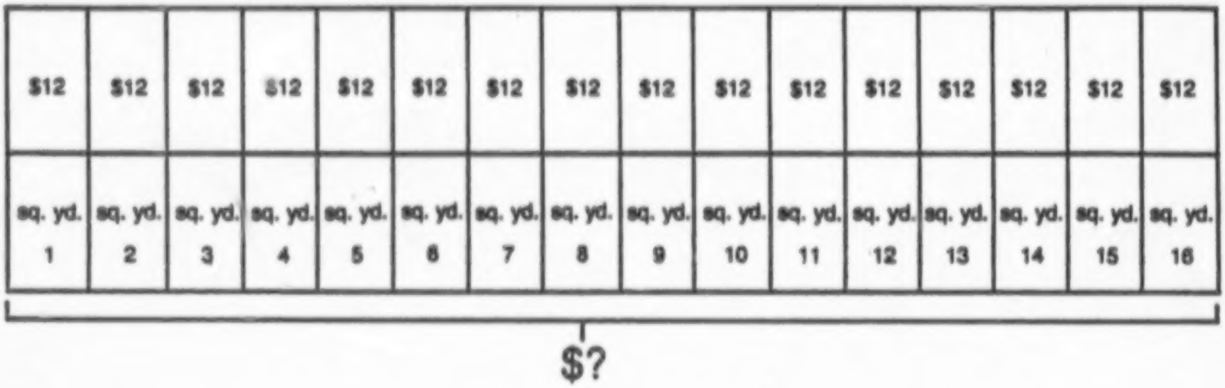

$12
$12
$12
$12
$12
$12
$12
$12
$12
$12
$12
$12
$12
$12
$12
$12
sq. yd. 1
sq. yd. 2
sq. yd. 3
sq. yd. 4
sq. yd. 5
sq. yd. 6
sq. yd. 7
sq. yd. 8
sq. yd. 9
sq. yd. 10
sq. yd. 11
sq. yd. 12
sq. yd. 13
sq. yd. 14
sq. yd. 15
sq. yd. 16
$?

Fig. 14 A *rate 2* problem: Jane pays $162 for carpeting at $9 a square yard. How many square yards did she get?

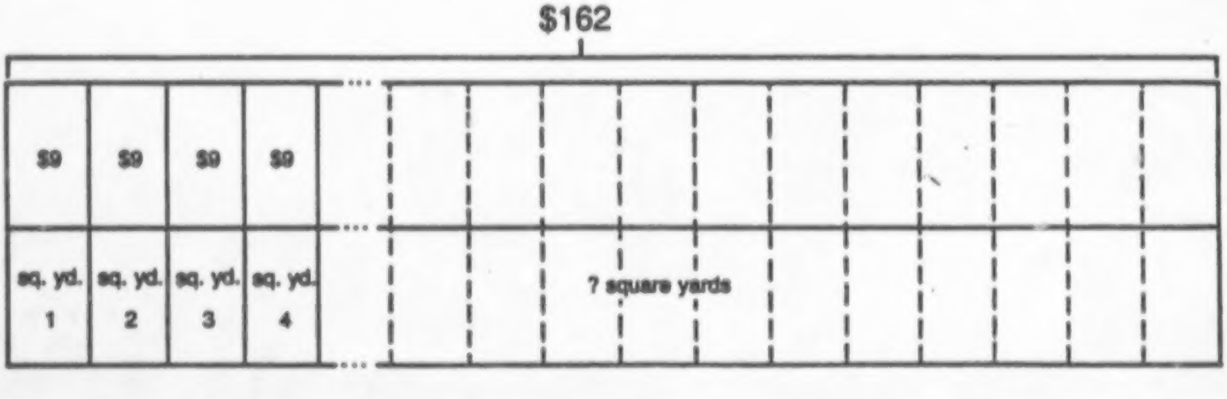
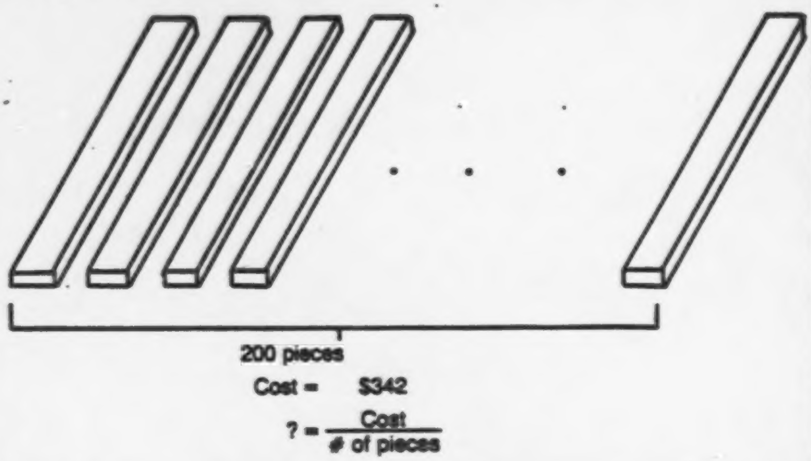

$162
$9
$9
$9
$9
sq. yd. 1
sq. yd. 2
sq. yd. 3
sq. yd. 4
? square yards

Fig. 15 A *rate 3* problem: Peter paid $342 for 200 eight-foot 2 x 4's. What was the cost in dollars of each 2 x 4?



- feet per second
- dollars per pound
- pounds per cubic foot
- gallons per minute
- cents per kilowatt hour
- parts per hundred

Children who are unable to think about rates and ratios will have difficulty doing these problems in any way other than substituting numbers into memorized formulas. Problems dealing with percentages are probably the best example of this difficulty.

Rate 1 problems result when the rate and the value of independent variable quantity are given (usually in units of measurement) and the value of the dependent variable, usually a total, must be found (fig. 13):

> Fred pays $12 a square yard for outdoor carpeting. How much will 16 square yards cost him?

The resulting application,

total cost
= cost/sq. yd. × number of sq. yd.
= $12/sq. yd. × 16 sq. yd. = $192,

is the easiest of the rate situations to use.

Rate 2 problems result when the rate and the value of the dependent variable are given and the value of the independent variable is sought (fig. 14):

> Jane pays $162 for carpeting at $9 a square yard. How many square yards does she get?

We have

$$\$162 = \$9/\text{sq. yd.} \times \boxed{}\ \text{sq. yd.}$$

or

$$\text{sq. yd.} = \frac{\$162}{\$9/\text{sq. yd.}} = \boxed{18}.$$

Rate 3 problems result when the values of the dependent and independent variables are given and the ratio or comparison rate is sought (fig. 15):

> Peter paid $342 for 200 eight-foot two-by-fours. What was the cost in dollars of each two-by-four?

We have

$$\$342 = \boxed{\$}/\text{board} \times 200\ \text{boards}$$

or

$$\$\ \text{cost/board} = \frac{\$342}{200\ \text{boards}} = \$1.71/\text{board}$$

Selection Problems

Among the most difficult problems are those that require multiplication. These belong to a more general group of selection problems.

Selection 1 problems involve simple ordered pairs where the choice sets for each element of the ordered pair are given and the number of ordered pairs possible is sought. The pairs are ordered in the sense that one choice set is associated with one element and a second choice set with the other. No ordering occurs in the writing or selection. In the following example, (skirt, sweater) is not different from (sweater, skirt). See figure 16.

> Amy has 3 sweaters with different patterns. She also has 5 different skirts. How many outfits consisting of a sweater and a skirt are possible?

The pairs can be determined from a matrix (table 1) or from a "factor tree." Either way, multiplication is used: 3 × 5 = 15 outfits.

Selection 2 problems result when one choice set and the number of pairs are given and the other choice set is sought. These problems are similar to selection 1 problems.

Table 1
A Matrix to Record the Pairs in Figure 16

	Skirts				
Sweaters	1	2	3	4	5
A	A, 1	A, 2	A, 3	A, 4	A, 5
B	B, 1	B, 2	B, 3	B, 4	B, 5
C	C, 1	C, 2	C, 3	C, 4	C, 5

Selection 3 problems involve triples, quadruples, or other extended n-tuples ($n > 2$) and the choice sets for each place in the n-tuple.

> Frank has 5 sport coats, 3 vests, and 5 pairs of trousers, all of which are color compatible. How many different outfits consisting of a sport coat, vest, and pair of trousers are in his wardrobe?

Here a 3-tuple must be formed (sport coat, vest, trousers) where ordering is not important. Finding the total number of 3-tuples uses the multiplication principle: 5 × 3 × 5 = 75.

Selection 4 problems give the number of n-tuples and the sizes of all but one choice set, which is sought. An example follows:

> Frank can make 24 different outfits consisting of a sport coat, vest, and trousers. He has 3 sport coats and 4 pairs of trousers. How many vests does Frank have?

Fig. 16 A selection 1 problem: Amy has 3 sweaters with different patterns. She also has 5 skirts of different colors. How many outfits, consisting of a sweater and a skirt, are possible?

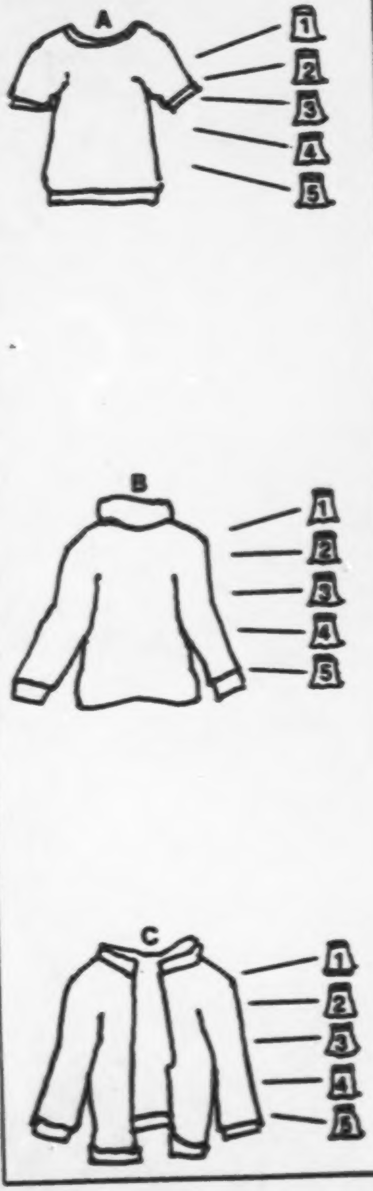


Fig. 17 Ceramic tiles can be used to link the repeated-addition idea of multiplication to area: Make 4 rows of 6 tiles each. How many tiles are used?

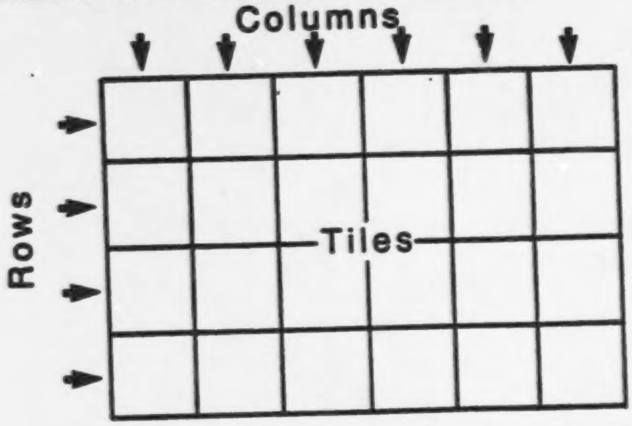


Table 2
Change Problems

Problem title	Sample problem	Characteristics
(Change 1) Repeated addition	Fred has 3 boxes with 4 cars in each box. How many cars does Fred have?	Initial (smaller) set sizes and change number known; question about final (larger) set.
(Change 2) Repeated subtraction (measurement)	Jean had 12 cookies. She gave 3 cookies to each of her friends. How many of her friends got cookies?	Initial (larger) set and final (smaller) set sizes known; question about change number.
(Change 3) Partitioning into equal sets	Paul had 24 marbles that he gave away to 4 friends. Each friend received the same number of marbles. How many marbles did each friend get?	Initial (larger) set and change numbers known; question about the size of final (smaller) sets.

This is a two-step problem: first multiply and then divide, or successively divide.

The selection group of problems involves the multiplication principle or one aspect of what Piaget calls combinatorial reasoning—the ability to consider the effect of several variables simultaneously. Selection 1 problems can be thought of as cells in a matrix. The thinking needed to solve them is similar to that used to solve area problems, such as being given two dimensions and finding the area and being given the area and one dimension and finding the other dimension.

Overview

If students are going to apply multiplication and division to everyday situations, they must have experience with materials that represent these different situations.

The change situations that involve joining and separating can be introduced with materials that can be joined, separated, and arranged. Unifix cubes can be used to illustrate repeated additions and repeated subtractions as well as measurements. Ceramic tiles can also be used to link the idea of repeated addition to area (fig. 17). The measurement concept of division can also be introduced with tiles. The following kinds of questions can be asked:

- Given 24 tiles, how many rows can be made with 4 tiles in each row?
- Make 4 rows of 6 tiles each. How many tiles are used?

Beans and paper cups can be used to give experience with the partition interpretation of division as well as to the repeated-addition and repeated-subtraction interpretations of multiplication and division. Some examples

Table 3
Compare Problems

Problem title	Sample problem	Characteristics
Compare 1	Joellen has 3 pairs of sandals. She has 4 times as many pairs of shoes. How many pairs of shoes does she have?	Referent set and many-to-one correspondence known; question about the comparison set.
Compare 2	Irene has 30 pennies. She has 5 times as many pennies as Pat has. How many pennies does Pat have?	Comparison set and many-to-one correspondence known; question about the referent set.
Compare 3	Donald has 5 marbles. Peter has 15 marbles. Peter has how many times as many marbles as Donald?	Comparison set and referent set given; question about kind of (many-to-one) correspondence.
Compare 4	Bonnie has 16 white blouses and 4 colored blouses. Her colored blouses are what (fractional) part of her white blouses?	Comparison set and referent set given; question about kind of (one-to-many) correspondence.
Compare 5	Our class has 16 boys and 12 girls: There are how many times as many boys as girls?	Comparison set and referent set given; question about the (many-to-many) correspondence.
Compare 6	Our class has 16 boys and 12 girls. The girls are what (fractional) part of the boys?	Comparison set and referent set given; question about many-to-many correspondence.
Compare 7	Fred has 25 baseball cards. He has 5/4 as many cards as Jim has. How many baseball cards does Jim have?	Comparison set and many-to-many correspondence given; question about referent set.
Compare 8	Erica has 25 stickers. Peggy has 4/5 as many stickers as Erica. How many stickers does Peggy have?	Referent set and many-to-many correspondence given; question about comparison set.

Table 4
Selection Problems

Problem title	Sample problem	Characteristics
Selection 1	Paula has 3 kinds of cheese and 2 kinds of sausage. How many different cheese-and-sausage pizzas can she make?	Number given from which to select for each pair element; question about number of pairs possible.
Selection 2	Frank makes 18 different cheese-and-sausage pizzas. He has 6 kinds of cheese. How many kinds of sausage does he have?	Number in one choice set and number of pairs given; question about number in other choice set.
Selection 3: extended *n*-tuple	Dave has 3 different-sized sets of wheels, 4 kinds of bodies, and 3 different motors. How many different cars with wheels, a body, and a motor can he put together?	Number given from which to choose for each portion in *n*-tuple; question about number of *n*-tuples possible.
Selection 4: extended *n*-tuple	Dave has 3 different-sized sets of wheels and 4 kinds of bodies; he can make 96 different cars with wheels, bodies, and motors. How many different kinds of motors does he have?	Number given from which to choose for all but one position in *n*-tuple and also number of *n*-tuples; question about remaining position.

Table 5
Rate Problems

Problem title	Sample problem	Characteristics
Rate 1	Lisa buys 18 cans of polish at \$0.72 per can. What is the total cost?	Given the rate and the independent variable value; question is about the dependent variable.
Rate 2	Peter buys a suit on sale. The price, after a 25% discount, is \$90. What was the original price?	Given the rate and the dependent variable value; question is about the independent variable.
Rate 3	Corrine runs 200 meters in 72 seconds. What is her average speed in meters per second?	Given the values of the dependent and independent variables; question is about the rate.

are the following:

- Given 21 beans, put 3 beans in cups until the beans are gone. How many cups did you use?
- Given 35 beans, put an equal number of beans into each of 5 cups. How many beans are in each cup?
- Given 4 cups, put 5 beans in each cup. How many beans were needed?

The *ratio comparison* situations can be introduced with two different shapes, two different colors of chips or cubes, or any other materials that can be put into sets and compared using the multiplication- and division-related questions in the examples.

The *selection* ideas can be introduced best with colored cubes or several geometric shapes in different colors, forming pairs and triples of these materials. Subsequently using situations that involve items from the students' experience, such as stickers, pizza toppings, clothing, and record labels, can help children apply these basic ideas of multiplication to the real world.

Rate problems should probably be introduced after establishing the idea of a constant rate of change in two related variables. This introduction must be done slowly and carefully and timed to the stage of cognitive development of the students. The demands are primarily on the proportional-reasoning capability of the students.

Introducing problems involving such relationships as *distance* = *time* × *rate*, *cost* = *cost/unit* × *units*, and *percentage* = *percent* × *base* should be within the more general context of rate of change. Otherwise students may substitute values into formulas without understanding the processes involved.

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BLACK LINE MASTERS

for many of the

WORKSHEETS

and

RECORDING FORMS

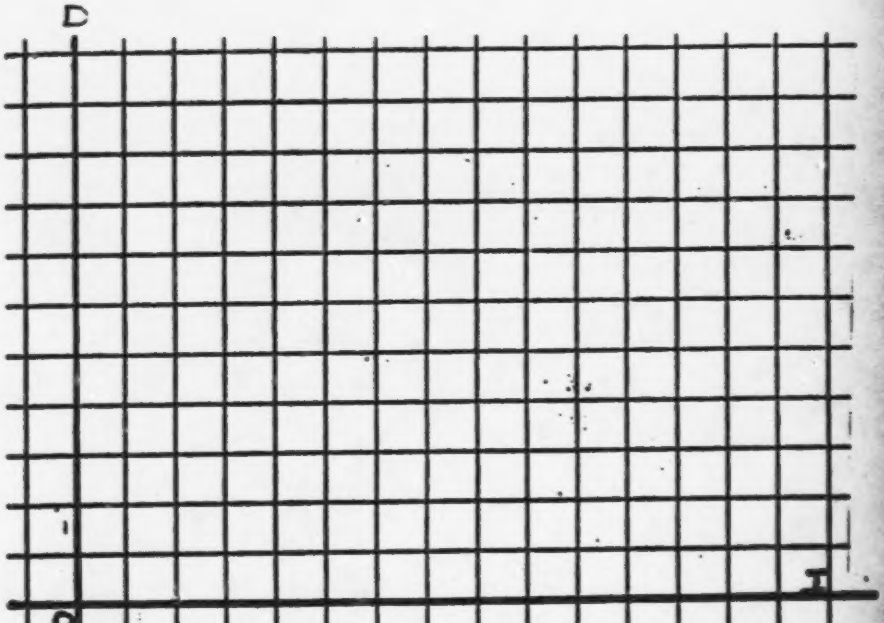
Mathematician:______________________________________

"I completed each table of values for D and I, found the constant change in D and the rule and graphed this."

TABLE

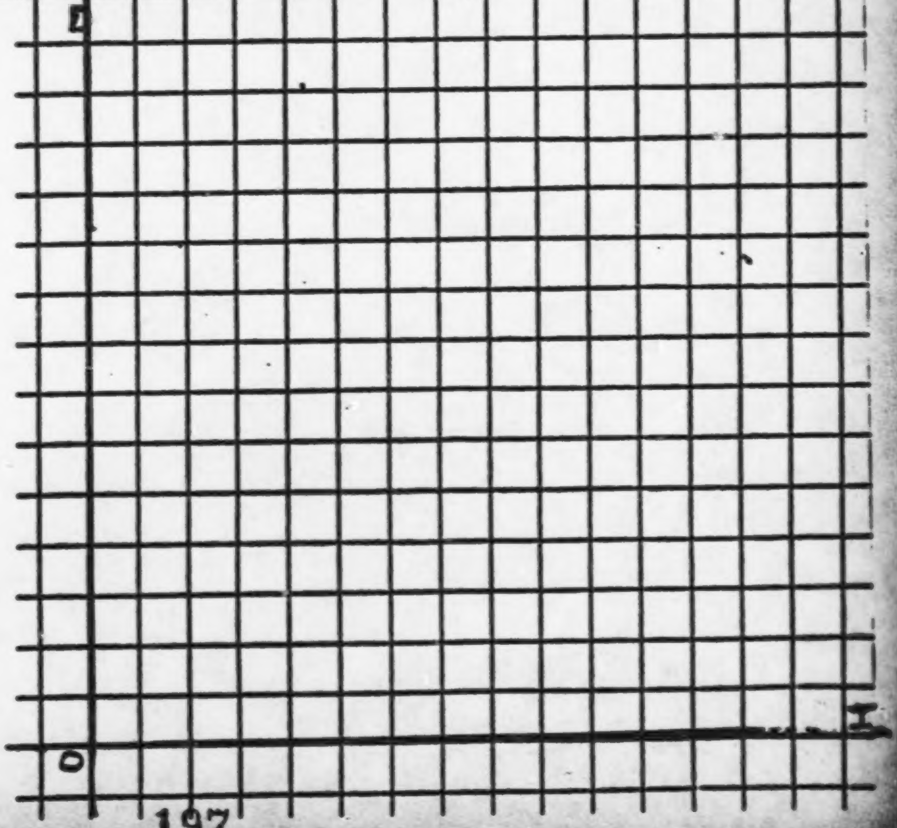
GRAPH

I	D	Change in D



RULE:

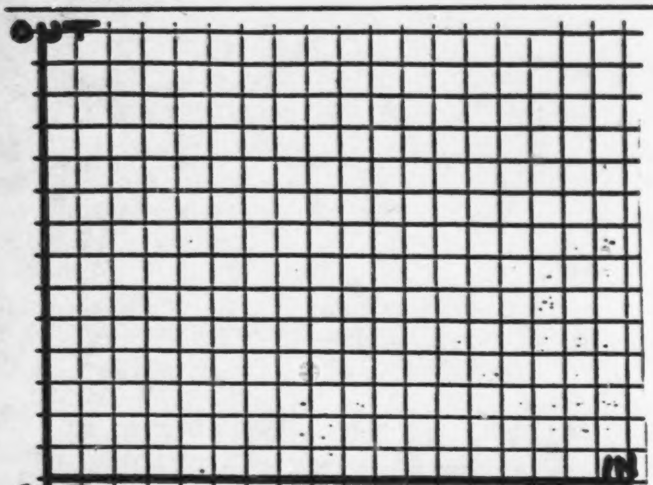
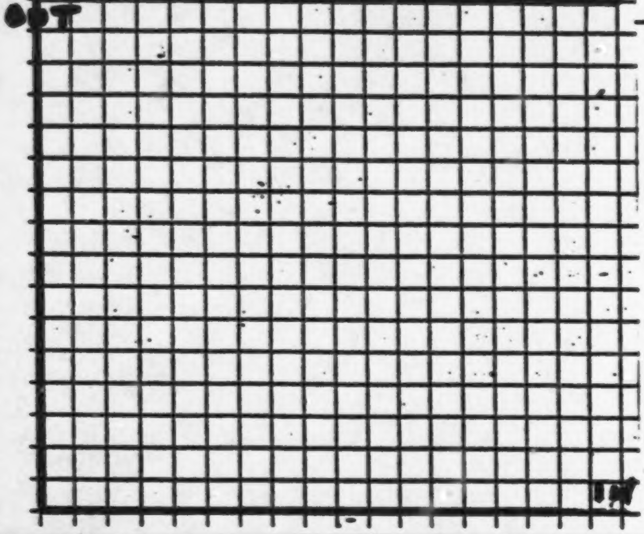
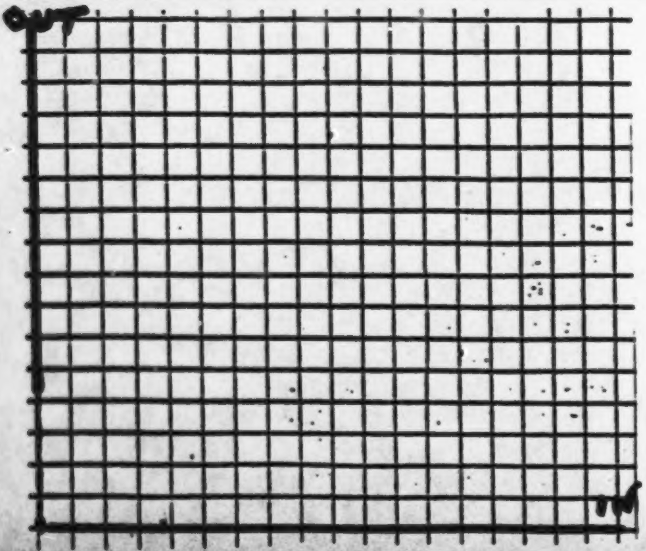
I	D	Change in D



Rule:

Mathematician: ______________________________

"I found rules for the graphs given."

GRAPH	RULE
	**RULE:** ____________________
	**RULE:** ____________________
	**RULE:** ____________________

10388

Mathematician:____

"I changed the products in sums or differences as indicated."

Product	Sum or Difference	Shown as equal
Example: 3 (7 + 5)	3 (7) + 3 (5)	3(12)=36; 3(7)+3(5)=21+15=36

Mathematician:______________________________

"I completed the following showing how a product is expressed as a sum or difference of products."

PRODUCT	SUM OR DIFFERENCE
Example: $5(7+3)$	$5 \times 7 + 5 \times 3 = 35 + 15 = 50$
$6(4-2)$	$6 \times 4 - 6 \times 2 = 24 - 12 = 12$

Mathematician:____

"I showed the equality between adding two arrays and a single array in these exercises."

Given array	Two arrays	Number Sentences
Example: 5 x 8 = 40	5 (7 + 1) 5 (6 + 2) 5 (5 + 3) 5 (4 + 4)	5 (7+1)=5x7+5x1=35+5=40 5 (6+2) = 5x 6+5x2 = 30 + 10 =40 5(5+3) = 5x.5 +5x3 = 25+15=40 5(4+4) = 5x4+5x4 =20+20=40

Mathematician: ____________________

"I first estimated these lengths, then measured them in BOTH inches and centimeters."

LENGTHS	MEASURE	TO THE NEAREST WHAT PART OF A UNIT?
	inches centimeters	
	inches centimeters	

6.12.85.1

Mathematician:____

"I used a graph paper transparency to find the areas of these shapes."

SHAPE	AREA

6.12.89.2

Mathematician:____

"I used the graph paper transparency to find the areas of these triangles."

Mathematician: ______________________________

"I shaded the bars to show the fractions given. Then I joined the parts, found the difference between the parts and compared the parts two ways."

JOINED __________ + __________ = __________

DIFFERENCE __________ = __________ - __________

COMPARED
LARGER TO SMALLER __________ + __________ = __________

COMPARED
SMALLER TO LARGER __________ = __________ + __________

JOINED __________ + __________ = __________

DIFFERENCE __________ = __________ - __________

COMPARED
LARGER TO SMALLER __________ + __________ = __________

COMPARED
SMALLER TO LARGER __________ = __________ + __________

110188

Mathematician:____

"I did the multiplications, drew a picture of the hundreds, ten and ones, regrouped where needed and completed the computation form."

Multiplication	Base Ten Blocks	Expanded form

11788

Mathematician:____

"I used base ten blocks to do the HORIZONTAL multiplications. I recorded the partial products found."

MULTIPLICATION	FINDING THE FOUR PARTIAL PRODUCTS	FINAL PRODUCT
	1. 2. 3. 4. Total:	
	1. 2. 3. 4. Total:	
	1. 2. 3. 4. Total:	
	1. 2. 3. 4. Total:	
		6.12.89.4

Mathematician:____

"I did these divisions by places, using base ten blocks. I put the base ten blocks into a rectangle as I subtracted them in the division."

DIVISION	PICTURE	ANSWER
		6.12.89.3

Mathematician:____

"I found the PERIMETERS for these shapes."

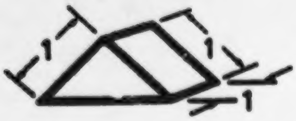
Mathematician:____

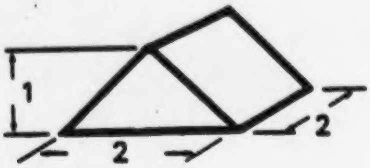
"I found the AREAS or missing parts of these rectangles and parallelograms.
I wrote the AREA INSIDE and labelled the missing parts.

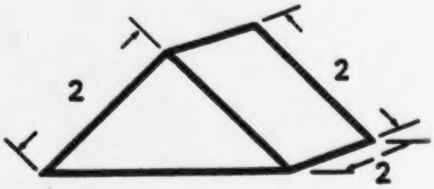
Example:

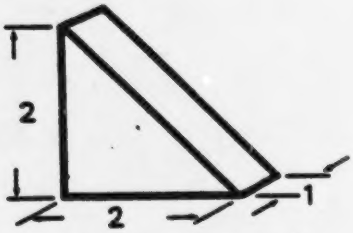
11888

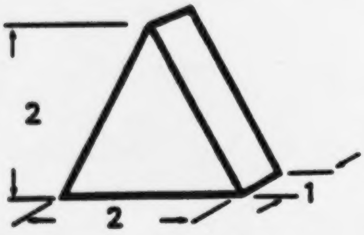
TRIANGULAR PRISMS

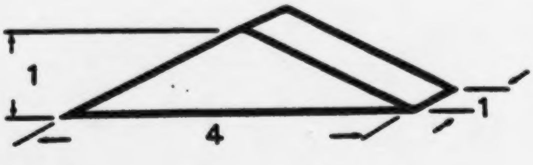


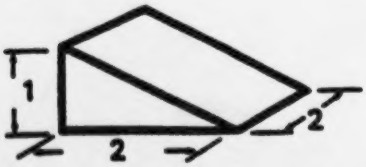


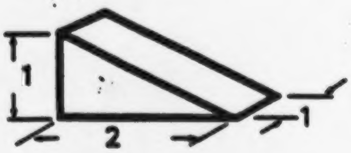


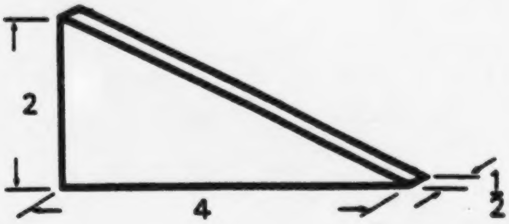


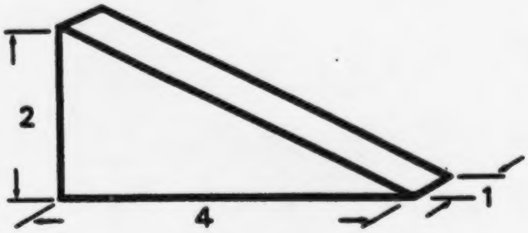


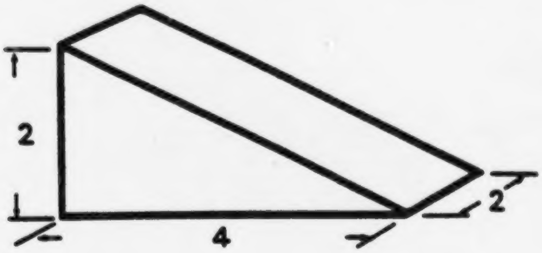


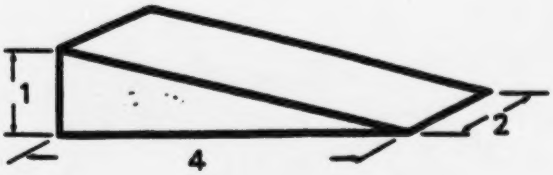


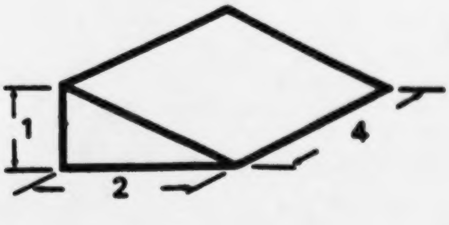




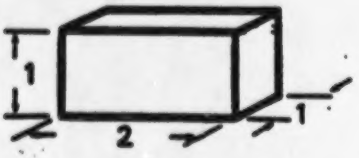


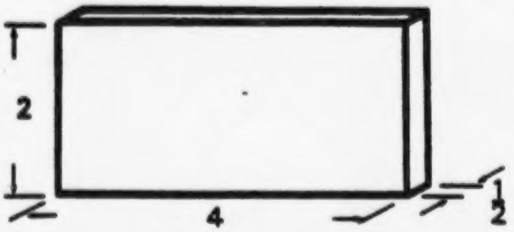


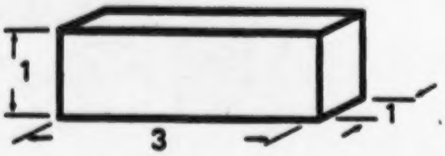


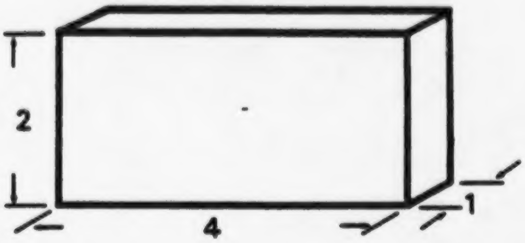


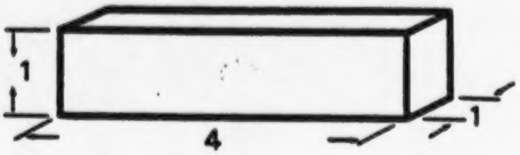
RECTANGULAR PRISMS

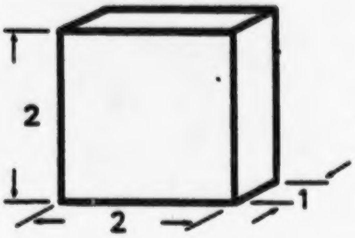


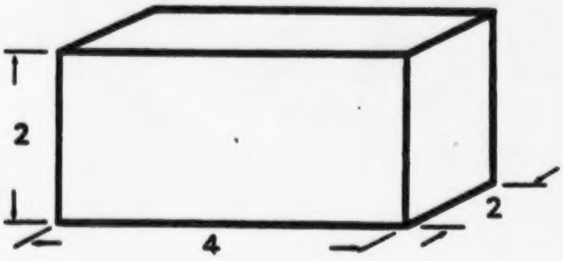


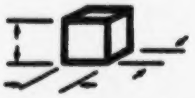




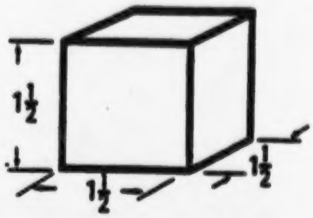




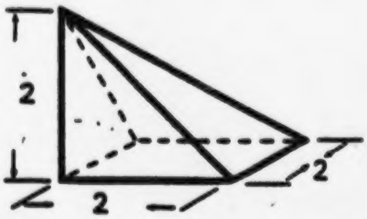


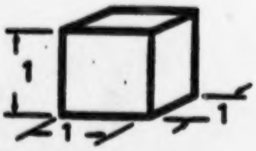


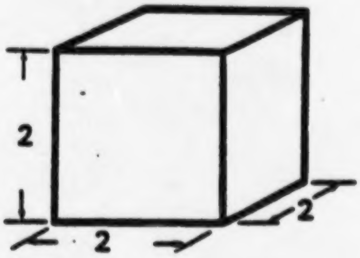
C-1 $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$



PYRAMID







Mathematician: ______________________________

"I made the geoblocks shown out of blocks like the one given."

Mathematician:____

"One geojacket has lines of symmetry drawn. I drew lines of symmetry in the others given where possible."

Mathematician:____

"I added numbers to both sides or subtracted numbers from both sides to find the number for ☐ ."

OPEN SENTENCE	WHAT I DID

Mathematician: ____

"I wrote open sentences to show the board, removed or added + and − chips as needed to solve for ☐."

Equality Board	Open Sentence	The number for ☐
=		
=		
=		

Mathematician:____

"I did these multiplications using base ten blocks and wrote the results in horizontal form. I did all carrying before recording."

COMPUTATION	STEPS	NUMBER SENTENCE
	TENS X TENS TENS X ONES ONES X TENS ONES X ONES	________Hundreds ________Tens ________Tens ________Ones ________________
		________Hundreds ________Tens ________Tens ________Ones ________________
		________Hundreds ________Tens ________Tens ________Ones ________________
		________Hundreds ________Tens ________Tens ________Ones ________________

Mathematician:______________________________

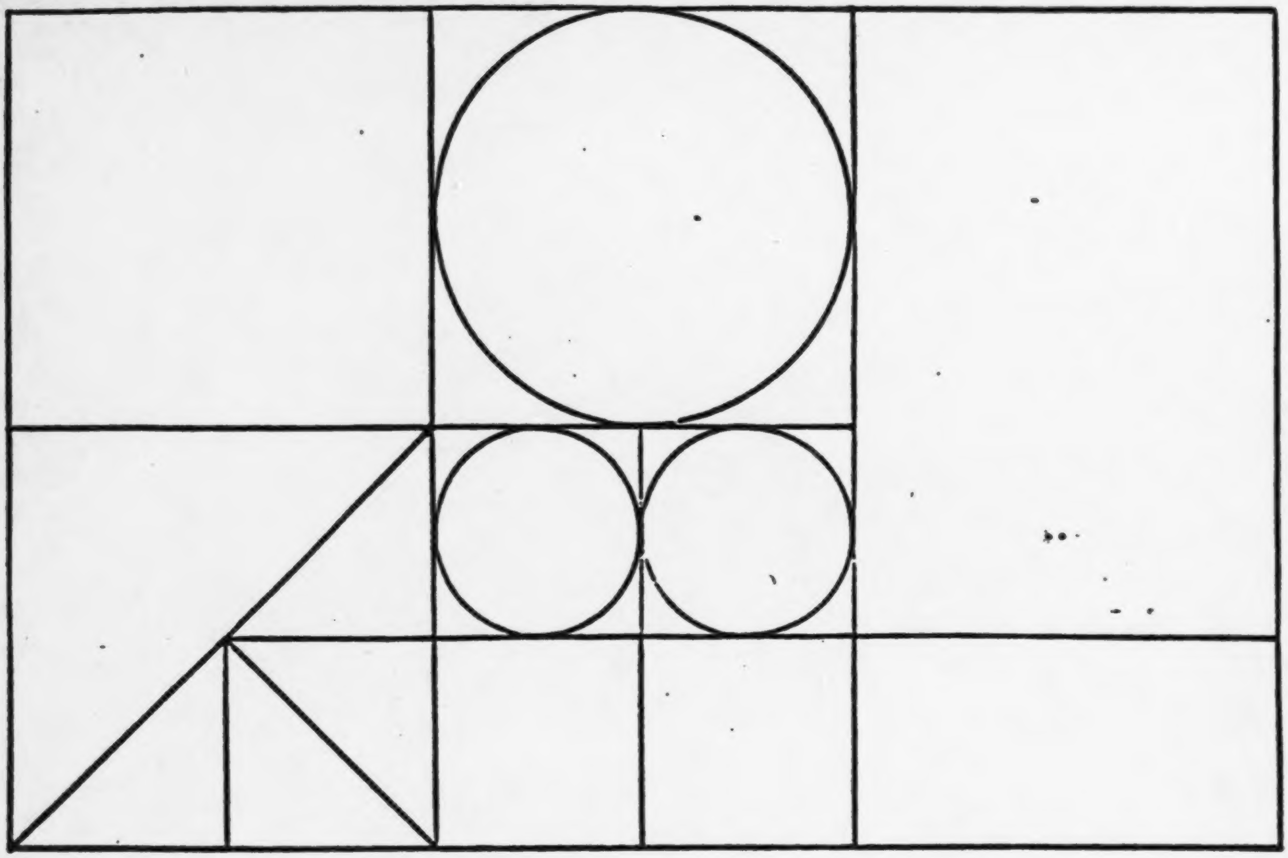
"I used base ten blocks at first to do these additions in horizontal form (going along a line like we read)."

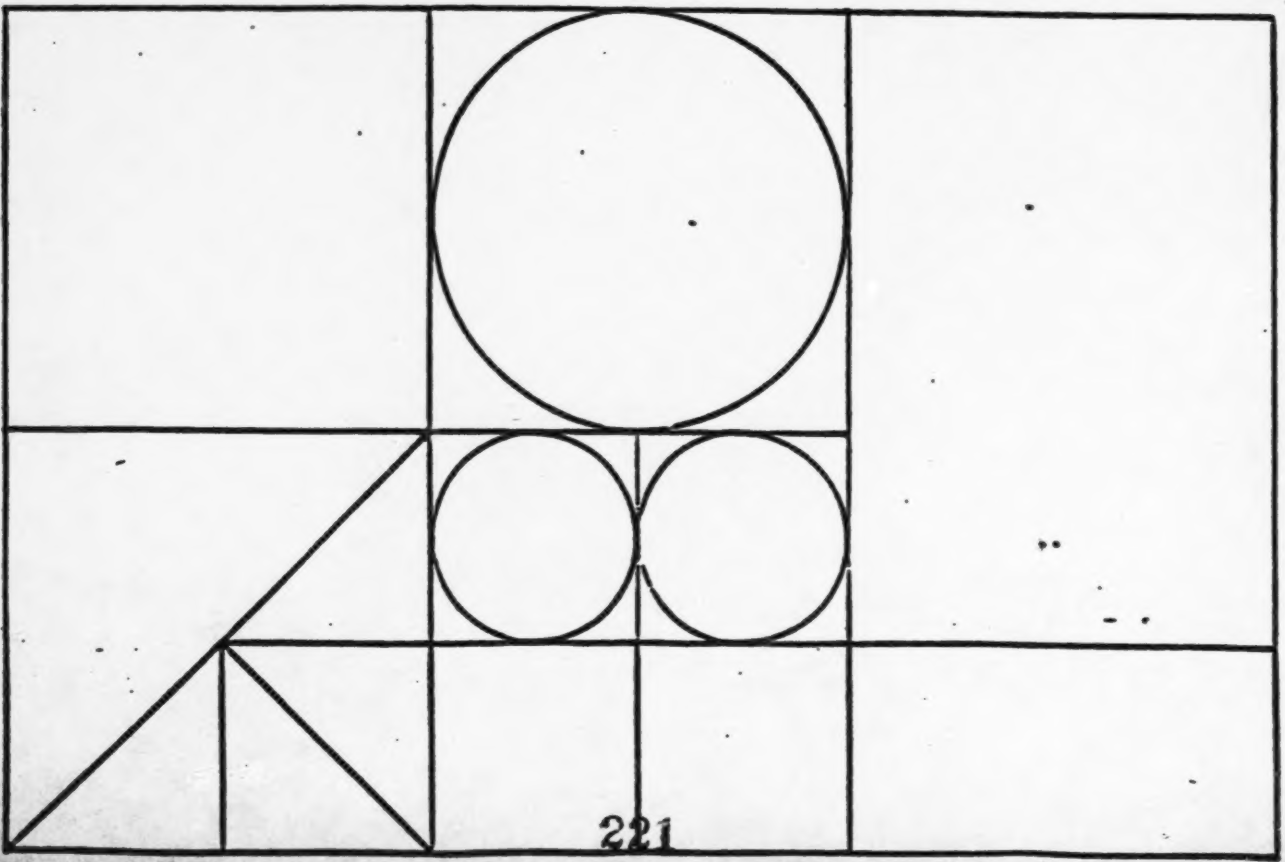
COMPUTATION	STEPS	NUMBER SENTENCES
	TENS AND TENS ONES AND ONES	________Tens ________Ones ______________
		________Tens ________Ones ______________
		________Tens ________Ones ______________
		________Tens ________Ones ______________
		________Tens ________Ones ______________
		________Tens ________Ones ______________

Mathematician____

"For each group of 3 numbers, I wrote a story using these. The circled number answers the question in each story problem."

NUMBERS GIVEN	STORY PROBLEM





"NOT" LABEL CARDS

NOT BLUE

NOT GREEN

NOT RED

NOT TRIANGLE

NOT SQUARE

NOT RECTANGLE

NOT CIRCLE

NOT LARGE

NOT SMALL

LABEL CARDS

RED	**CIRCLE**
BLUE	**RECTANGLE**
GREEN	**LARGE**
TRIANGLE 	**SMALL**
SQUARE 	

Mathemtician: ______________________________

"I wrote statements TRUE of the given set of attribute blocks, and found a piece that would make one of these FALSE."

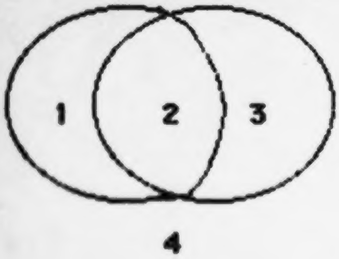
TRUE STATEMENTS	PIECE TO MAKE STATEMENT FALSE

1258.3

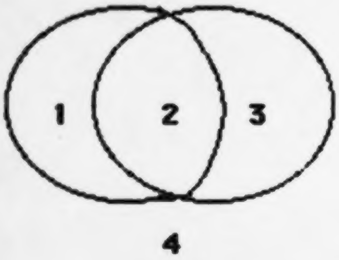
Mathematician:____

"I sorted the blocks into each region. I wrote a description of the blocks in each of regions 1, 2, 3 and 4."

REQUIREMENTS **DESCRIPTIONS**



1.
2.
3.
4.



1.
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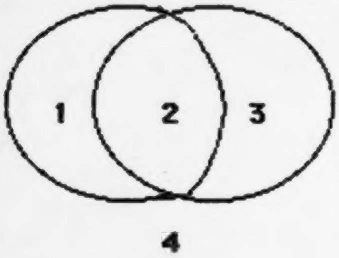

1.
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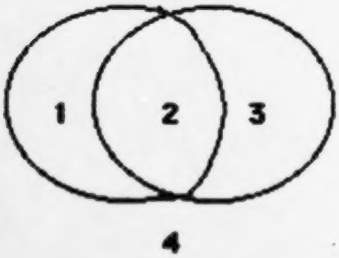
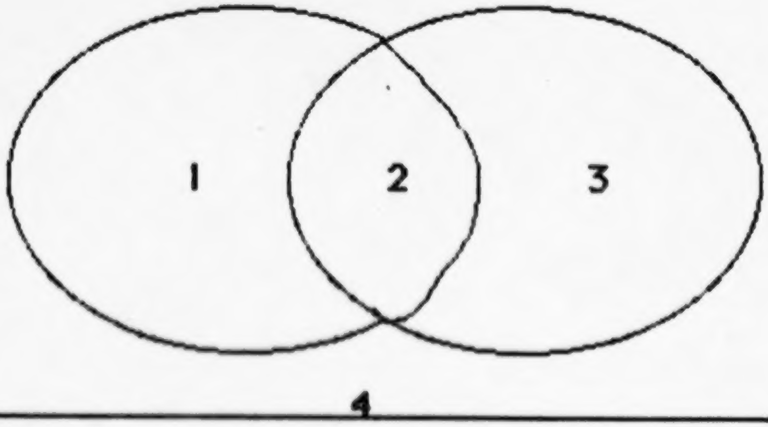
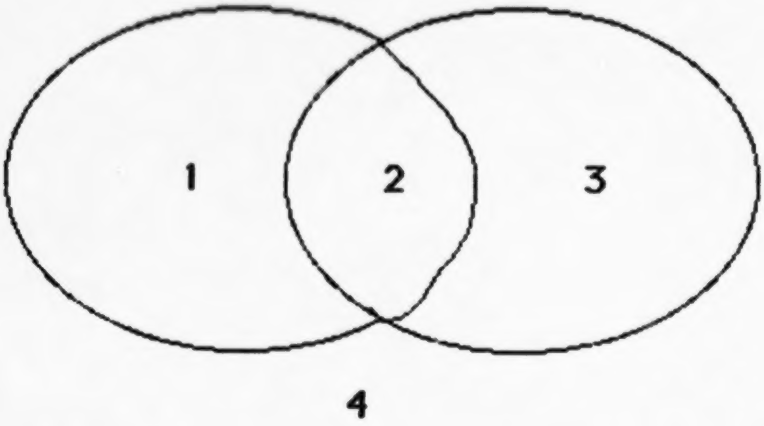


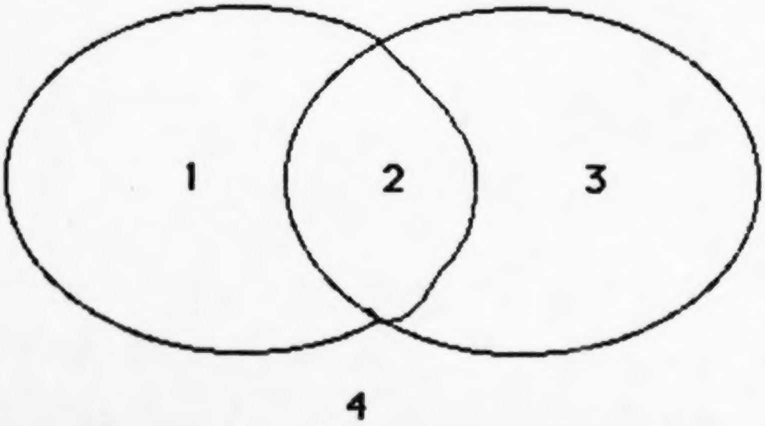
1.
2.
3.
4.

Mathematician:____

"I sorted the pieces according to each set of requirements and drew pictures of the pieces in each place."







Mathematician:____

"I wrote IF-THEN sentences for the collections given that are TRUE. For each TRUE statement, I found a piece that would make it FALSE when added to the collection."

PIECES GIVEN	TRUE STATEMENTS	PIECE TO MAKE GIVEN TRUE STATEMENT FALSE

MEATS

FAMILY PAK' BONELESS
COUNTRY STYLE RIBS
$1 19 LB.

FAMILY PAK
PORK STEAK
89¢ LB.

REGULAR 3 LB. PKG.
GROUND BEEF
89¢ LB.

WILSON'S CORN KING
SKINLESS FRANKS
69¢ 12 OZ. PKG.

WILSON'S CORN KING
SLICED BACON
99¢ 1 LB. PKG.

KLEMENT'S FRES
BRATWURST
$1 59 LB.

CORN DOGS .... LB.	$1 59
CHICKEN NIBBLERS .... LB.	$2 49
HERRING FILLETS	$2 19
OUR HOMEMADE BY THE PIECE SYLTA .... LB.	$2 29
3 VARIETIES SLICED THIN MEATS	2/99¢
MR. TURKEY GROUND TURKEY	99¢

PRODUCE

NAVEL
ORANGES
10 FOR 99¢

FRESH CRISP CABBAGE .... LB.	25¢
SEEDLESS RED GRAPES .... LB.	99¢
DELICIOUS KIWI FRUIT	3/$1
RUBY RED GRAPEFRUIT	$1 99
FRESH CUCUMBERS	4/$1
GREEN PEPPERS	3/$1

GRADE AA
LARGE EGGS
38¢ 1 DOZ. CTN.
WITH COUPON BELOW

DOVE LIQUID
DISH DETERGENT
$1 59 48 OZ. BTL.
WITH COUPON BELOW

SURF LIQUID
LAUNDRY DETERGENT
$4 99 1 GAL. BTL.
WITH COUPON BELOW

BAKED BEANS 2/$1 16 OZ. CAN

SPAGHETTI SAUCE 99¢

APPLE SAUCE 99¢

PILLSBURY **CAKE MIX** 69¢

SPAGHETTI 99¢ 2 LB. PKG.

KRISPY CRACKERS 79¢ 1 LB. PKG.

HOT COCOA MIX	99¢
CLAM CHOWDER	$1 19
OATMEAL	$1 49
BROWN GRAVY MIX	3/$1
CHILI POWDER	$1 99
CHILI SAUCE	99¢
WORCESTERSHIRE SAUCE	89¢
[illegible]	79¢
[illegible]	99¢
POPCORN	4/$1 19
CHEESE PUFFS	99¢
[illegible]	2/99¢
PEANUT BUTTER	$1 19
[illegible]	$1 29
APPLE JUICE	$1 19
[illegible]	$1 69

DAIRY FOODS

VELVET SOFT
SPREAD MARGARINE
69¢ 2 LB. BOWL

KEMPS CHIP DIP	3/$1
FOOD CLUB CREAM CHEESE	78¢
FOOD CLUB SLICED MOZZARELLA	89¢
FOOD CLUB GRATED PARMESAN	88¢
KRAFT CHEESE SPREAD	79¢

FROZEN FOODS

OLD ORCHARD
ORANGE JUICE
99¢ 12 OZ. CAN

MR. DEES GOLDEN [illegible] .	$1 29
BANQUET SALISBURY STEAK OR SLICED TURKEY ENTREES .	$1 99
MR. DEES SHRIMP CURLS ... 11 OZ.	$2 49
KEMPS QUALITY CHEK'D ICE CREAM	$1 99
KEMPS FROZEN YOGURT	$2 49

COUPON
GRADE AA
LARGE EGGS
38¢ 1 DOZ. CTN.

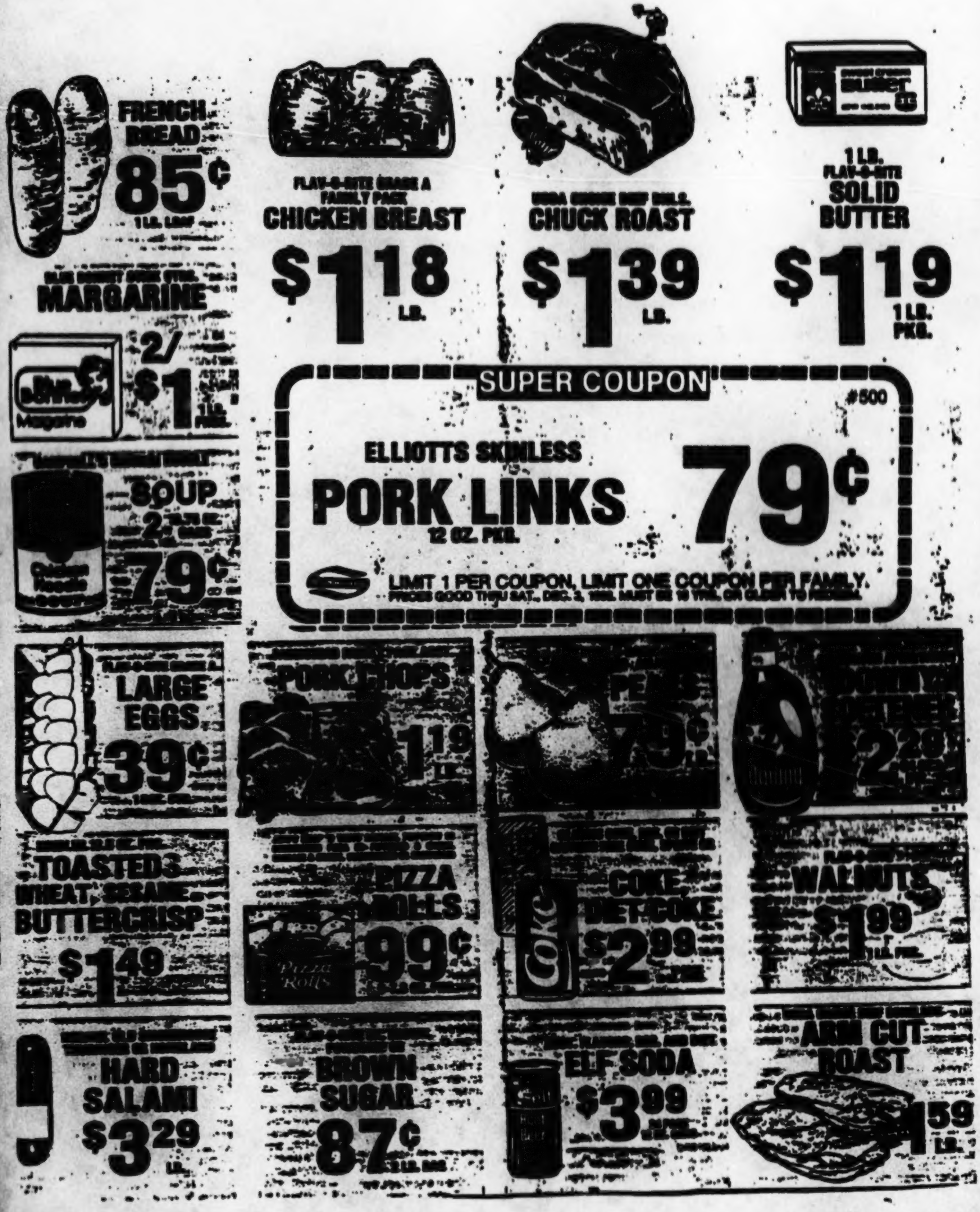
COUPON
DOVE LIQUID
DISH DETERGENT
$1 59 48 OZ. BTL.

COUPON
SURF LIQUID
LAUNDRY DETERGENT
$4 99 1 GAL. BTL.

BUY 2 GET 1 FREE
Jerky Snacks for Dogs

DOG FOOD
$1 00 OFF ANY 4 LB. BAG

BUY THREE CANS
Puss'n Boots Supreme
CANNED CAT FOOD
GET ONE CAN FREE



FRENCH BREAD
85¢
1 LB. LOAF

[illegible]
MARGARINE
2/$1
1 LB. PKG.

[illegible]
SOUP
2 [illegible]
79¢

FLAV-O-RITE GRADE A
FAMILY PACK
CHICKEN BREAST
$1 18
LB.

[illegible]
CHUCK ROAST
$1 39
LB.

1 LB.
FLAV-O-RITE
SOLID BUTTER
$1 19
1 LB. PKG.

SUPER COUPON
#500
ELLIOTTS SKINLESS
PORK LINKS
12 OZ. PKG.
79¢
LIMIT 1 PER COUPON, LIMIT ONE COUPON PER FAMILY.
PRICES GOOD THRU SAT., DEC. 3, [illegible] MUST BE 18 YRS. OR OLDER TO REDEEM.

[illegible]
LARGE EGGS
39¢

PORK CHOPS
1 19
LB.

PEARS
79¢
LB.

DOWNY SOFTENER
$2 29

[illegible]
TOASTED WHEAT, SESAME, BUTTERCRISP
$1 49

[illegible]
PIZZA ROLLS
99¢

[illegible]
COKE, DIET COKE
$2 99

[illegible]
WALNUTS
$1 99

[illegible]
HARD SALAMI
$3 29
LB.

[illegible]
BROWN SUGAR
87¢
2 LB. BAG

[illegible]
ELF SODA
$3 99

[illegible]
ARM CUT ROAST
1 59
LB.

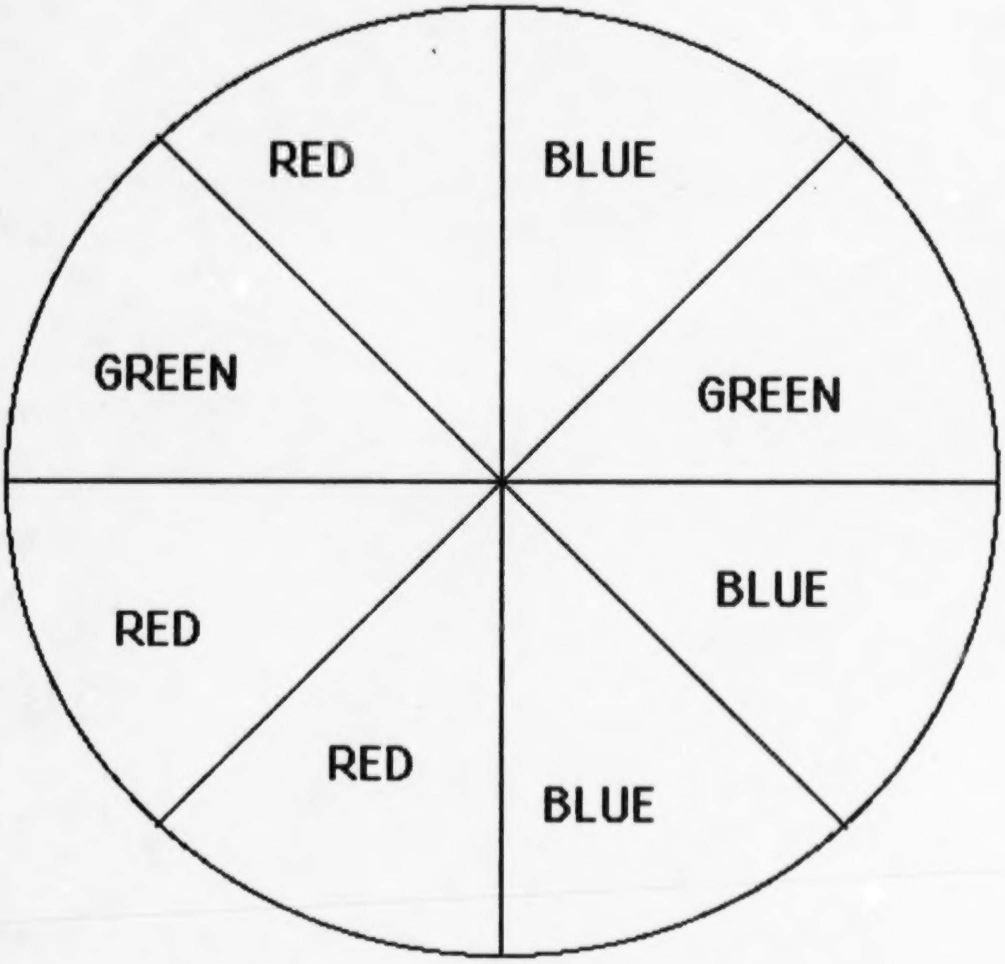
Mathematician:________________________________

"I drew geojackets for the block shown."

SPINNER TALLIES

RED	BLUE	GREEN

SPINNER BASE TEMPLATE


RED
BLUE
GREEN
GREEN
RED
BLUE
RED
BLUE

Make the pointer from cardboard. Use a paper fastener and a small washer to fasten it to the center of the spinner base.

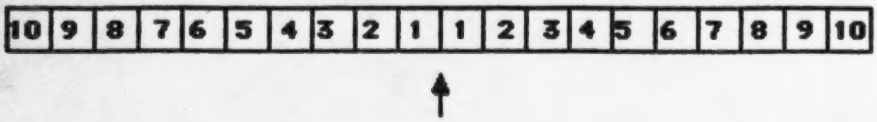
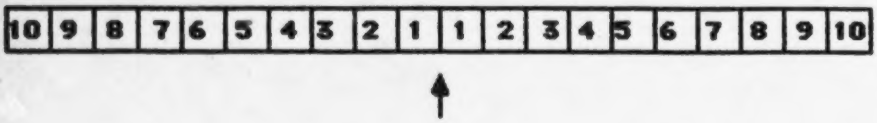
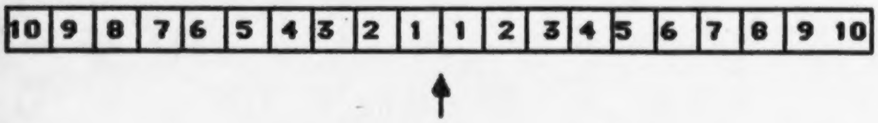
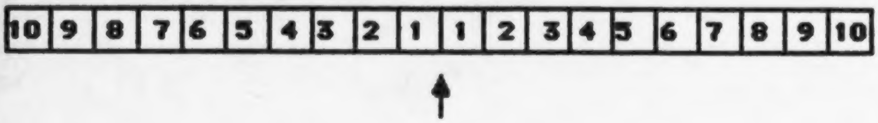
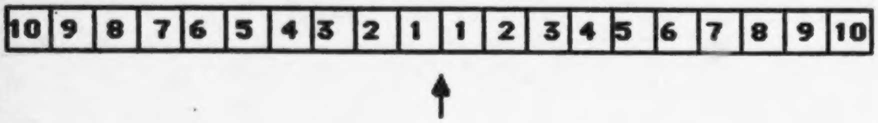
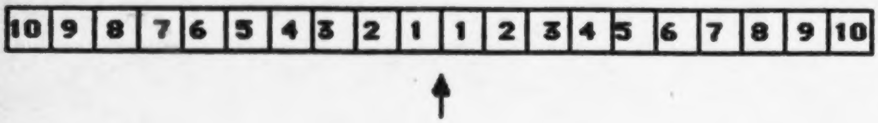
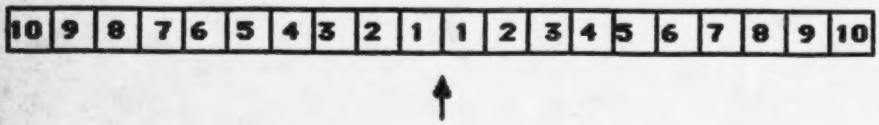
	TALLIES	TOTALS
HEADS		
TAILS		
TOTALS		50

Fraction of Heads $\frac{}{50}$

Fraction of Tails $\frac{}{50}$

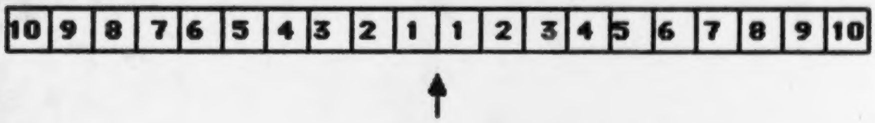
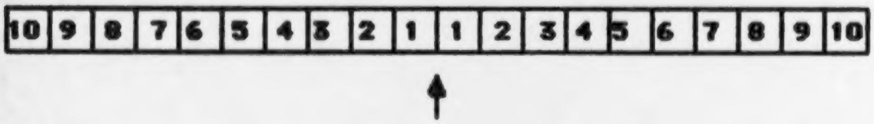
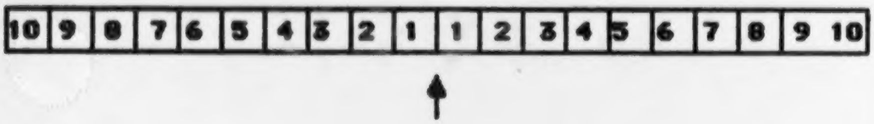
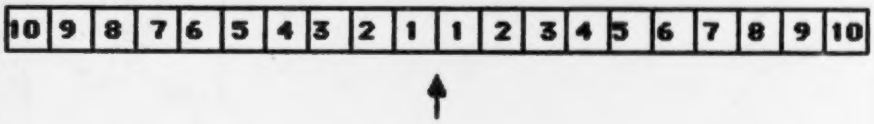
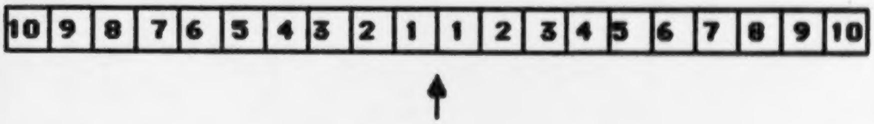
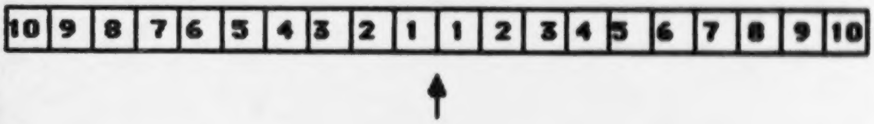
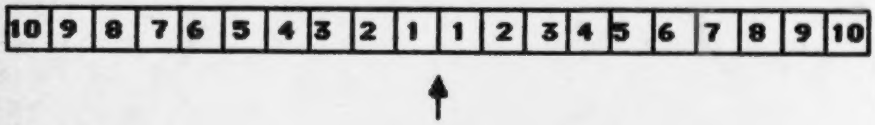
Mathematician: ______________________________

"For each balance beam shown I found the weight needed to balance it or the distance the weights given had to be placed to balance it."

BEAM	WEIGHT NEEDED	NUMBER AT WHICH TO PLACE WEIGHT
 10 9 8 7 6 5 4 3 2 1 1 2 3 4 5 6 7 8 9 10		
 10 9 8 7 6 5 4 3 2 1 1 2 3 4 5 6 7 8 9 10		
 10 9 8 7 6 5 4 3 2 1 1 2 3 4 5 6 7 8 9 10		
 10 9 8 7 6 5 4 3 2 1 1 2 3 4 5 6 7 8 9 10		
 10 9 8 7 6 5 4 3 2 1 1 2 3 4 5 6 7 8 9 10		
 10 9 8 7 6 5 4 3 2 1 1 2 3 4 5 6 7 8 9 10		
 10 9 8 7 6 5 4 3 2 1 1 2 3 4 5 6 7 8 9 10		

Mathematician: ____

"For each balance beam shown I found the weight needed to balance it or the distance the weights given had to be placed to balance it."

BEAM	WEIGHT NEEDED	NUMBER AT WHICH TO PLACE WEIGHT
 10 9 8 7 6 5 4 3 2 1 1 2 3 4 5 6 7 8 9 10		
 10 9 8 7 6 5 4 3 2 1 1 2 3 4 5 6 7 8 9 10		
 10 9 8 7 6 5 4 3 2 1 1 2 3 4 5 6 7 8 9 10		
 10 9 8 7 6 5 4 3 2 1 1 2 3 4 5 6 7 8 9 10		
 10 9 8 7 6 5 4 3 2 1 1 2 3 4 5 6 7 8 9 10		
 10 9 8 7 6 5 4 3 2 1 1 2 3 4 5 6 7 8 9 10		
 10 9 8 7 6 5 4 3 2 1 1 2 3 4 5 6 7 8 9 10		

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